STA 291
Fall 2009

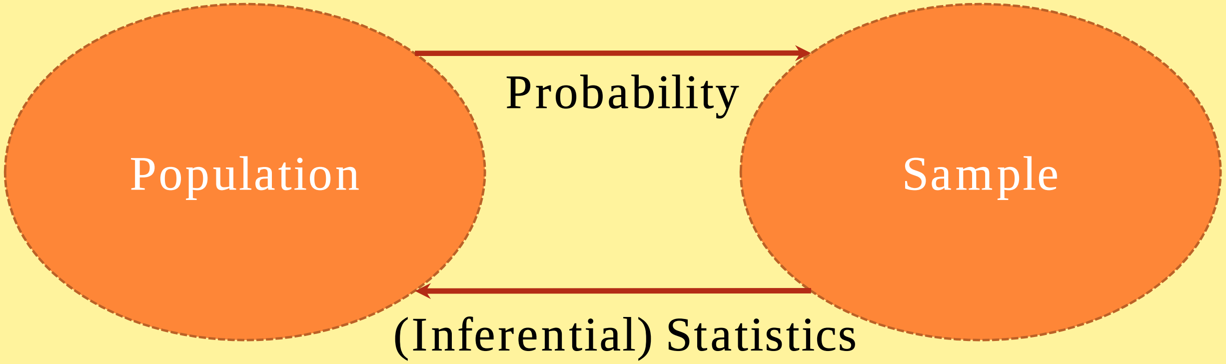
LECTURE 15
THURSDAY, 15 OCTOBER

Preview of Coming Attractions

- **5 Probability**

Chapter 5: Probability

- Abstract but necessary because this is the mathematical theory underlying all statistical inference



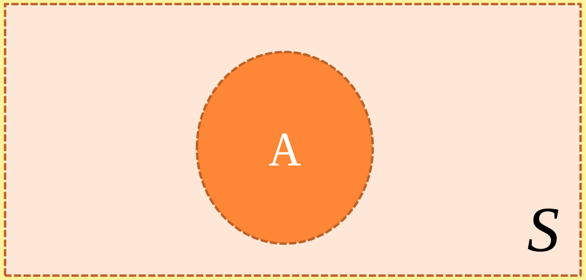
- Fundamental concepts that are very important to understanding *Sampling Distribution*, *Confidence Interval*, and *P-Value*
- Our goal for Chapter 5 is to learn the rules involved with assigning probabilities to events

Probability: Basic Terminology

- **Experiment**: Any activity from which an outcome, measurement, or other such result is obtained.
- **Random (or Chance) Experiment**: An experiment with the property that the outcome cannot be predicted with certainty.
- **Outcome**: Any possible result of an experiment.
- **Sample Space**: The collection of all possible outcomes of an experiment.
- **Event**: A specific collection of outcomes.
- **Simple Event**: An event consisting of exactly one outcome.

Complement

- Let A denote an event.
- The **complement** of an event A: All the outcomes in the sample space S that do not belong to the event A. The complement of A is denoted by A^c



Law of Complements: $P\left(A^c\right) = 1 - P\left(A\right)$

Example: If the probability of getting a "working" computer is 0.7, what is the probability of getting a defective computer?

Union and Intersection

- Let A and B denote two events.

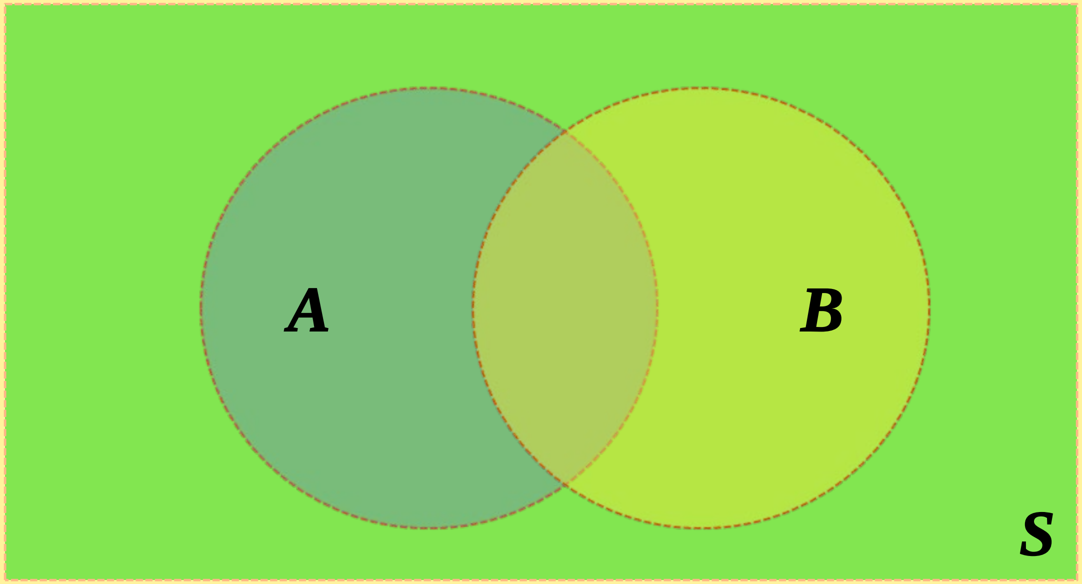
- The **union** of two events: All the outcomes in S that belong to at least one of A or B. The union of A and B is denoted by $A \cup B$

- The **intersection** of two events: All the outcomes in S that belong to both A and B. The intersection of A and B is denoted by $A \cap B$

Additive Law of Probability

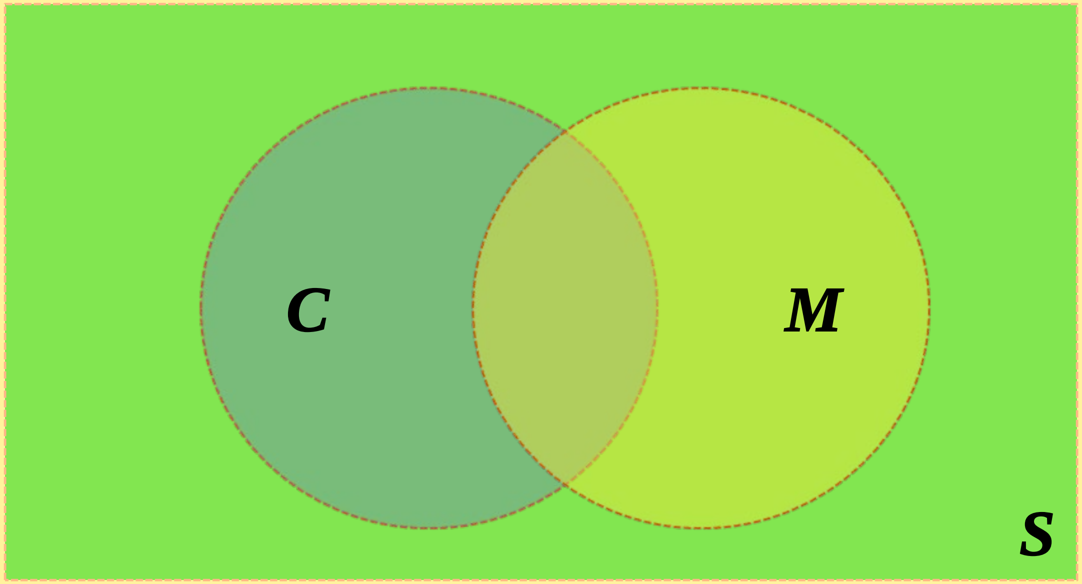
- Let A and B be *any* two events in the sample space S. The probability of the union of A and B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



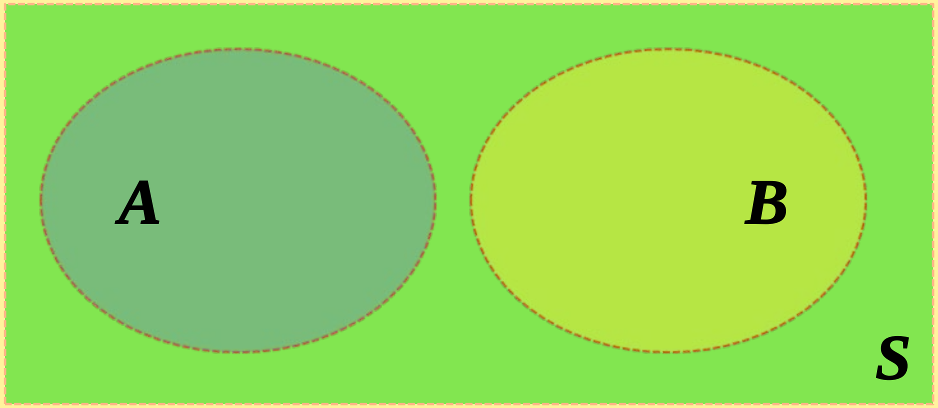
Additive Law of Probability

Example: At a large University, all first-year students must take chemistry and math. Suppose 85% pass chemistry, 88% pass math, and 78% pass both. Suppose a first-year student is selected at random. What is the probability that this student passed at least one of the courses?



Disjoint Sets

- Let *A and B denote two events.*
- **Disjoint (mutually exclusive) events**: *A* and *B* are said to be *disjoint* if there are no outcomes common to both *A* and *B.*
- The notation for this is written as $A \cap B = \{ \ \} = \phi$
- Note: The last symbol denotes the null set or the empty set.



Assigning Probabilities to Events

- The probability of an event is a value between 0 and 1.

- In particular:

– 0 implies that the event will never occur

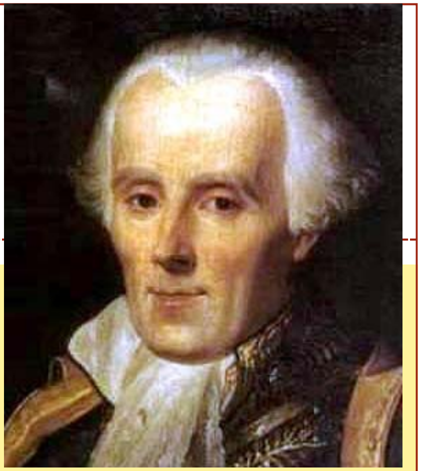
– 1 implies that the event will always occur

- How do we assign probabilities to events?

Assigning Probabilities to Events

- There are different approaches to assigning probabilities to events
- Objective
  - **equally likely outcomes (classical approach)**
  - **relative frequency**
- Subjective

Equally Likely Approach (Laplace)



- The equally likely outcomes approach usually relies on symmetry/geometry to assign probabilities to events.
- As such, we do not need to conduct experiments to determine the probabilities.
- Suppose that an experiment has only n outcomes. The equally likely approach to probability assigns a probability of $1/n$ to each of the outcomes.
- Further, if an event A is made up of m outcomes, then $P(A) = m/n$.

Equally Likely Approach

- Examples:

1. Roll a fair die
   - The probability of getting "5" is 1/6
   - This does not mean that whenever you roll the die 6 times, you definitely get exactly one "5"

2. Select a SRS of size 2 from a population

Relative Frequency Approach (von Mises)

- The relative frequency approach borrows from calculus' concept of limit.

- Here's the process:
  1. Repeat an experiment *n times*.
  2. Record the number of times an event *A* occurs. Denote that value by *a*.
  3. Calculate the value a/n

Relative Frequency Approach

- We could then define the probability of an event *A* in the following manner:

$$\text{Prob}(A) = \lim_{n \to \infty} \frac{a}{n}$$

- Typically, we can't can't do the "*n* to infinity" in real-life situations, so instead we use a "large" n and say that

$$\text{Prob}(A) \approx \frac{a}{n}$$

Relative Frequency Approach

- What is the formal name of the device that allows us to use "large" n?
- Law of Large Numbers:
  - As the number of repetitions of a random experiment increases,
  - the chance that the relative frequency of occurrences for an event will differ from the true probability of the event by more than any small number approaches 0.

Subjective Probability

- A subjective probability relies on a person to make a judgment as to how likely an event will occur.
- The events of interest are usually events that cannot be replicated easily or cannot be modeled with the equally likely outcomes approach.
- As such, these values will most likely vary from person to person.
- The only rule for a subjective probability is that the probability of the event must be a value in the interval [0,1]

Probabilities of Events

Let A be the event $A = \{o_1, o_2, ..., o_k\}$, where $o_1, o_2, ..., o_k$ are k different outcomes. Then

$$P(A)=P(o_1)+P(o_2)+\ldots+P(o_k)$$

Problem: The number on a license plate is any digit between 0 and 9. What is the probability that the first digit is a 3? What is the probability that the first digit is less than 4?

Conditional Probability & the Multiplication Rule

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) \neq 0$$

- Note: $P(A|B)$ is read as "the probability that A occurs given that B has occurred."
- Multiplied out, this gives *the multiplication rule:*

$$P(A \cap B) = P(B) \times P(A \mid B)$$

Multiplication Rule Example

- The multiplication rule:

$$P(A \cap B) = P(B) \times P(A \mid B)$$

- Ex.: A disease which occurs in .001 of the population is tested using a method with a false-positive rate of .05 and a false-negative rate of .05. If someone is selected and tested at random, what is the probability they are positive, and the method shows it?

Independence

- If events A and B are independent, then the events A and B have no influence on each other.
- So, the probability of A is unaffected by whether B has occurred.
- Mathematically, if A is independent of B, we write: $P(A|B) = \mathrm{P}(A)$

Multiplication Rule and Independent Events

Multiplication Rule for Independent Events: Let A and B be two independent events, then

$$P(A \cap B) = P(A)P(B).$$

Examples:

- Flip a coin twice. What is the probability of observing two heads?
- Flip a coin twice. What is the probability of getting a head and then a tail? A tail and then a head? One head?
- Three computers are ordered. If the probability of getting a "working" computer is 0.7, what is the probability that all three are "working" ?

Attendance Survey Question 15

- ***On a your index card:***
  - Please write down your name and section number
  - Today's Question: