STA 291
Fall 2009

LECTURE 16
TUESDAY, 20 OCTOBER

Probability

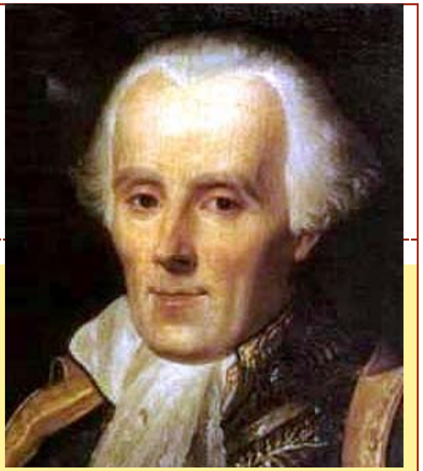
- **5 Probability**

- Suggested problems: 5.6, 5.9 – 5.14, 5.24, 5.33, 5.38,

Assigning Probabilities to Events

- There are different approaches to assigning probabilities to events
- Objective
  - **equally likely outcomes (classical approach)**
  - **relative frequency**
- Subjective

Equally Likely Approach (Laplace)



- The equally likely outcomes approach usually relies on symmetry/geometry to assign probabilities to events.
- As such, we do not need to conduct experiments to determine the probabilities.
- Suppose that an experiment has only *n* outcomes. The equally likely approach to probability assigns a probability of $1/n$ to each of the outcomes.
- Further, if an event *A* is made up of m outcomes, then $P(A) = m/n$.

Equally Likely Approach

- Examples:

1. Roll a fair die
   - The probability of getting “5” is 1/6
   - This does not mean that whenever you roll the die 6 times, you definitely get exactly one “5”

2. Select a SRS of size 2 from a population

Relative Frequency Approach (von Mises)

- The relative frequency approach borrows from calculus' concept of limit.

- Here's the process:
   1. Repeat an experiment *n times*.
   2. Record the number of times an event *A* occurs. Denote that value by *a*.
   3. Calculate the value a/n

Relative Frequency Approach

- We could then define the probability of an event *A* in the following manner:

$$\text{Prob}(A) = \lim_{n\to\infty} \frac{a}{n}$$

- Typically, we can't can't do the "*n* to infinity" in real-life situations, so instead we use a "large" n and say that

$$\text{Prob}(A) \approx \frac{a}{n}$$

Relative Frequency Approach

- What is the formal name of the device that allows us to use “large” *n*?
- Law of Large Numbers:
  - – As the number of repetitions of a random experiment increases,
  - – the chance that the relative frequency of occurrences for an event will differ from the true probability of the event by more than any small number approaches 0.

Subjective Probability

- A subjective probability relies on a person to make a judgment as to how likely an event will occur.
- The events of interest are usually events that cannot be replicated easily or cannot be modeled with the equally likely outcomes approach.
- As such, these values will most likely vary from person to person.
- The only rule for a subjective probability is that the probability of the event must be a value in the interval [0,1]

Probabilities of Events

Let A be the event $A = \{o_1, o_2, \ldots, o_k\}$, where $o_1, o_2, \ldots, o_k$ are k different outcomes. Then

$$P(A)=P(o_1)+P(o_2)+\ldots+P(o_k)$$

Problem: The number on a license plate is any digit between 0 and 9. What is the probability that the first digit is a 3? What is the probability that the first digit is less than 4?

Conditional Probability & the Multiplication Rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{provided } P(B) \neq 0$$

- Note: $P(A|B)$ is read as "the probability that A occurs given that B has occurred."
- Multiplied out, this gives *the multiplication rule:*

$$P(A \cap B) = P(B) \times P(A|B)$$

Multiplication Rule Example

- The multiplication rule:

$$P(A \cap B) = P(B) \times P(A \mid B)$$

- Ex.: A disease which occurs in .001 of the population is tested using a method with a false-positive rate of .05 and a false-negative rate of .05. If someone is selected and tested at random, what is the probability they are positive, and the method shows it?

Independence

- If events A and B are independent, then the events A and B have no influence on each other.

- So, the probability of A is unaffected by whether B has occurred.

- Mathematically, if A is independent of B, we write: $P(A|B) = \mathrm{P}(A)$

Multiplication Rule and Independent Events

Multiplication Rule for Independent Events: Let A and B be two independent events, then

$$P(A \cap B) = P(A)P(B).$$

Examples:

- Flip a coin twice. What is the probability of observing two heads?
- Flip a coin twice. What is the probability of getting a head and then a tail? A tail and then a head? One head?
- Three computers are ordered. If the probability of getting a "working" computer is 0.7, what is the probability that all three are "working" ?

Conditional Probabilities—Another Perspective

Example: Smoking and Lung Disease I

Joint Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12	.19	.31
Nonsmoker	.03	.66	.69
Column Totals	.15	.85	1.00

Conditional Probabilities—Another Perspective

Example: Smoking and Lung Disease I

Joint Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12	.19	.31
Nonsmoker	.03	.66	.69
Column Totals	.15	.85	1.00

Example: Smoking and Lung Disease II

Conditional Row Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12/.31 =.39	.19/.31 =.61	.31/.31 =1.00
Nonsmoker	.03/.69 =.04	.66/.69 =.96	.69/.69 =1.00
Smokers and Nonsmokers	.15	.85	1.00

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probabilities—Another Perspective

Example: Smoking and Lung Disease I

Joint Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12	.19	.31
Nonsmoker	.03	.66	.69
Column Totals	.15	.85	1.00

Example: Smoking and Lung Disease III

Conditional Column Probabilities	Lung Disease	Not Lung Disease	*Lung Disease and Not Lung Disease*
Smoker	.12/.15 =.80	.19/.85 =.22	.31
Nonsmoker	.03/.15 =.20	.66/.85 =.78	.69
Column Totals	.15/.15 =1.00	.85/.85 =1.00	1.00

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Terminology

- $P(A \cap B) = P(A,B)$ joint probability of A and B (of the intersection of A and B)

- $P(A|B)$ conditional probability of A given B

- $P(A)$ marginal probability of A

Attendance Survey Question 16

- ***On a your index card:***
  - Please write down your name and section number
  - Today's Question: