STA 291
Fall 2009

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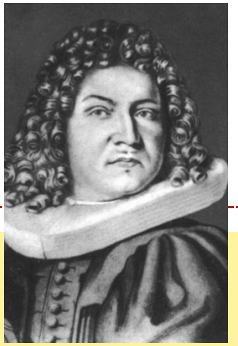
LECTURE 19
THURSDAY, 29 OCTOBER

Homework

- Graded online homework is due Saturday.
- Suggested problems from the textbook:

21.1 to 21.4,21.7 , 21.8, 21.10, and 21.11

- Exam 2 will be on November 4th Wed at 5pm to 7pm at Memorial Hall like Exam1.
- Make-up exam will be from 7:30pm to 9:30pm at the 8th floor of POT
- If you want to take the make-up exam send me email by this Sun.



Bernoulli Trial

- Suppose we have a single random experiment X with two outcomes: "success" and "failure."
- Typically, we denote "success" by the value 1 and "failure" by the value 0.
- It is also customary to label the corresponding probabilities as:

$$P(\text{success}) = P(1) = p \text{ and}$$

$$P(\text{failure}) = P(0) = 1 - p = q$$

- Note: $p + q = 1$

Binomial Distribution I

- Suppose we perform several Bernoulli experiments and they are all independent of each other.
- Let's say we do n of them. The value n is the **number of trials.**
- We will label these n Bernoulli random variables in this manner: $X_1, X_2, \ldots, X_n$
- As before, we will assume that the probability of success in a single trial is p, and that this probability of success doesn't change from trial to trial.

Binomial Distribution II

- Now, we will build a new random variable X using all of these Bernoulli random variables:

$$X = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i$$

- What are the possible outcomes of X?
- What is X counting?
- How can we find $P(X = x)$?

Binomial Distribution III

- We need a quick way to count the number of ways in which k successes can occur in n trials.
- Here's the formula to find this value:

$$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}, \text{ where } n! = n\cdot(n-1)\cdot\cdots\cdot 3\cdot 2\cdot 1 \text{ and } 0! \equiv 1$$

- Note: ${}_nC_k$ *is read as* "n choose k."

Binomial Distribution IV

- Now, we can write the formula for the binomial distribution:
- The probability of observing *x* successes in *n* independent trials is

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \text{ for } x = 0, 1, \cdots, n$$

under the assumption that the probability of success in a single trial is *p*.

Using Binomial Probabilities

Note: Unlike generic random variables where we would have to be given the probability distribution or calculate it from a frequency distribution, here we can calculate it from a mathematical formula.

Helpful resources (besides your calculator):

- Excel:

Enter	Gives
=BINOMDIST(4,10,0.2,FALSE)	0.08808
=BINOMDIST(4,10,0.2,TRUE)	0.967207

http://www.stattrek.com/Tables/Binomial.aspx

Binomial Probabilities

We are choosing a random sample of $n = 7$ Lexington residents—our random variable, C = number of Centerpointe supporters in our sample. Suppose, $p = P$ (Centerpointe support) ≈ 0.3. Find the following probabilities:

a) $P(C = 2)$

b) $P(C < 2)$

c) $P(C \leq 2)$

d) $P(C \geq 2)$

e) $P(1 \leq C \leq 4)$

What is the *expected* number of Centerpointe supporters, μ_C?

Center and Spread of a Binomial Distribution

- Unlike generic distributions, you don't need to go through using the ugly formulas to get the mean, variance, and standard deviation for a binomial random variable (although you'd get the same answer if you did):

$$\mu = np$$

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

Continuous Probability Distributions

- For continuous distributions, we can not list all possible values with probabilities
- Instead, probabilities are assigned to intervals of numbers
- The probability of an individual number is 0
- Again, the probabilities have to be between 0 and 1
- The probability of the interval containing all possible values equals 1
- Mathematically, a continuous probability distribution corresponds to a (density) function whose integral equals 1

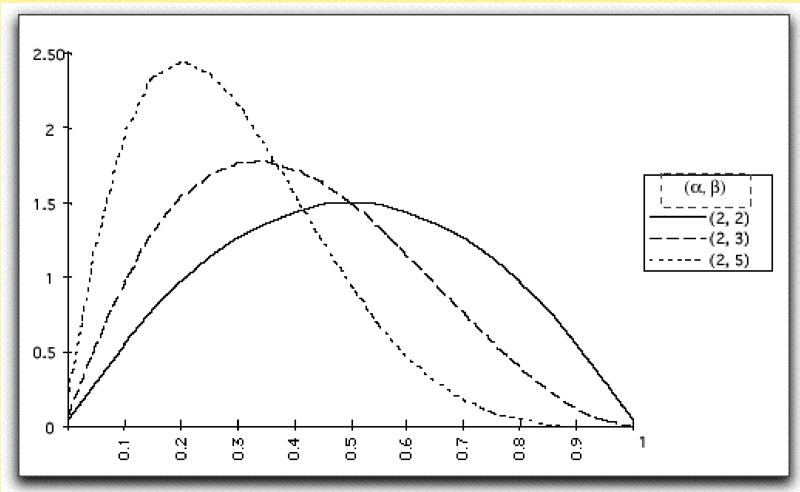
Continuous Probability Distributions: Example

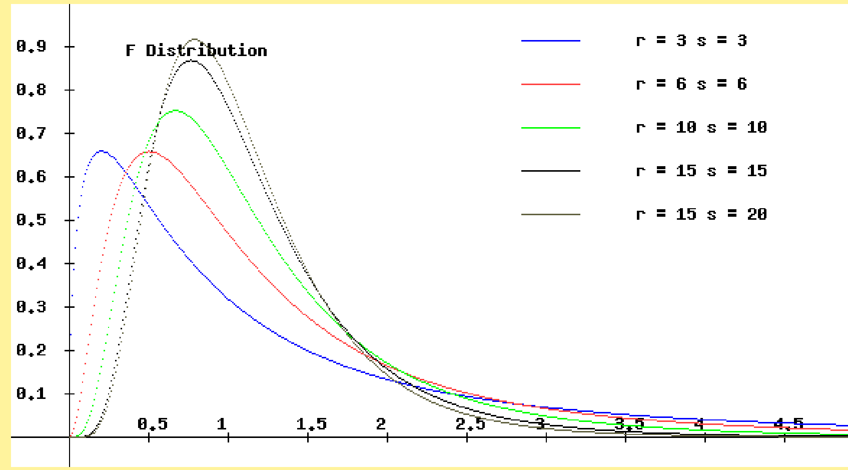
- Example: X=Weekly use of gasoline by adults in North America (in gallons)
- $P(6<X<9)=0.34$
- The probability that a randomly chosen adult in North America uses between 6 and 9 gallons of gas per week is 0.34
- Probability of finding someone who uses exactly 7 gallons of gas per week is 0 (zero)—might be *very close* to 7, but it won't be exactly 7.

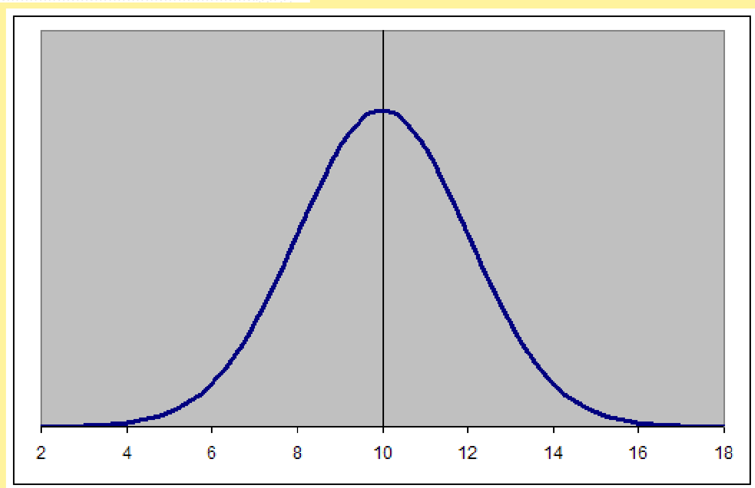
Graphs for Probability Distributions

- Discrete Variables:
  - Histogram
  - Height of the bar represents the probability
- Continuous Variables:
  - Smooth, continuous curve
  - Area under the curve for an interval represents the probability of that interval

Some Continuous Distributions







Attendance Question #19

Write your name and section number on your index card.

Today's question: