STA291
Fall 2009

LECTURE 26

TUESDAY, 24 NOVEMBER

Administrative Notes

- *This week's online homework due on the next Mon.*

- Practice final is posted on the web as well as the old final.
- *Suggested* Reading
  - Study Tools or Textbook Chapter 11

- *Suggested* problems from the textbook:

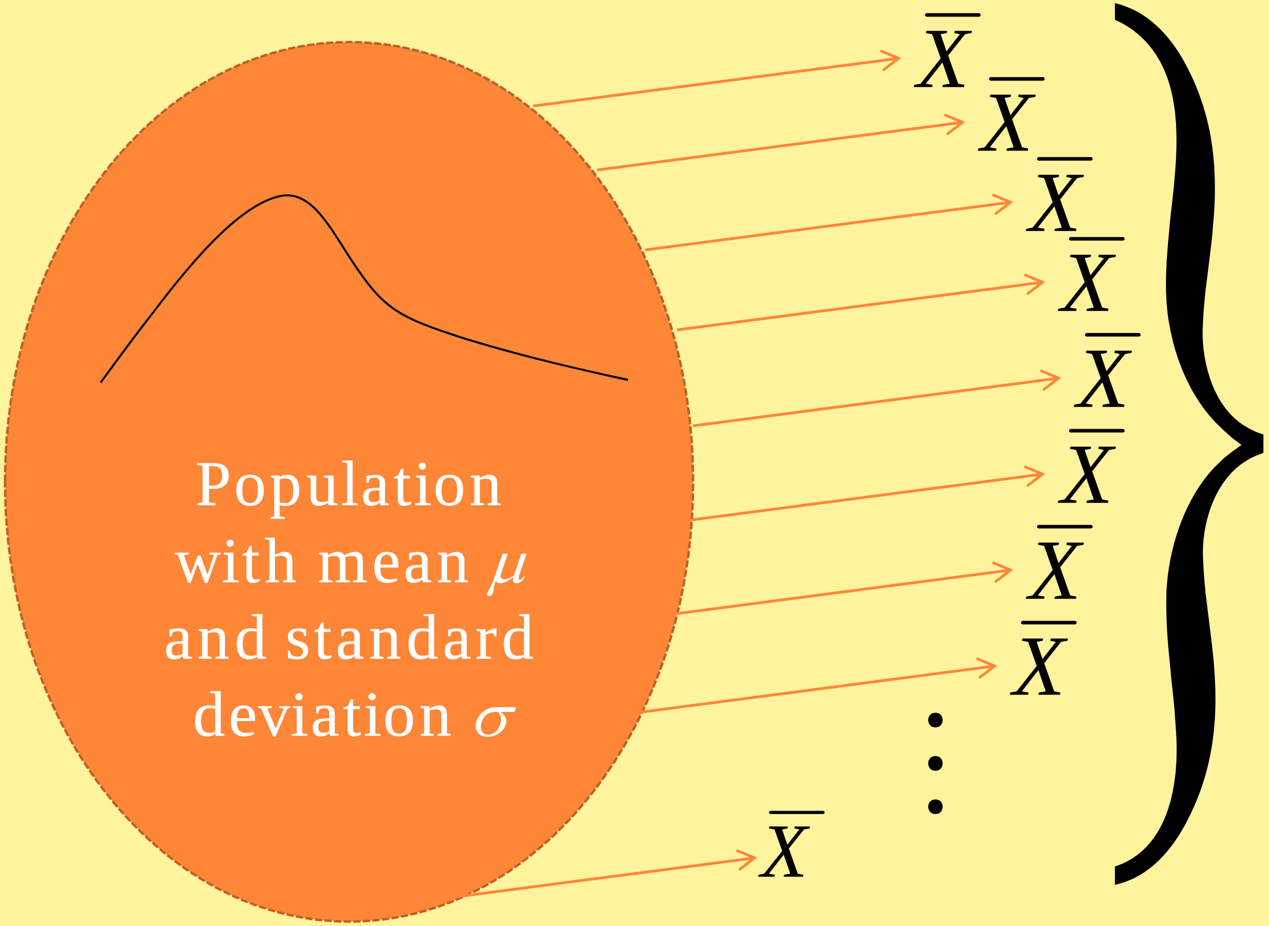
  11.1 – 11.6

Review on sampling distribution

Question "If we increase the sample size n the confidence interval decreases…. So why increasing the sample side is a good thing?"

Answer "this is because sampling distribution is distributed according to the normal distribution with the mean μ and the standard deviation $\frac{\sigma}{\sqrt{n}}$

Recall: Sampling Distributions



- If you repeatedly take random samples and calculate the sample mean each time, the distribution of the sample mean follows a pattern
- This pattern is the *sampling distribution*

Properties of the Sampling Distribution

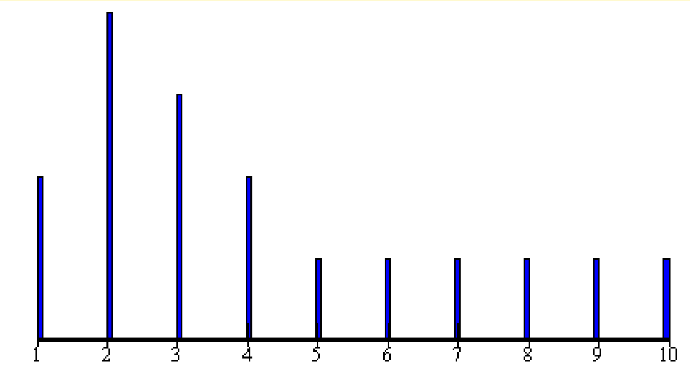
- Expected Value of the $\bar{X}$'s: μ.
- Standard deviation of the $\bar{X}$'s: $\dfrac{\sigma}{\sqrt{n}}$

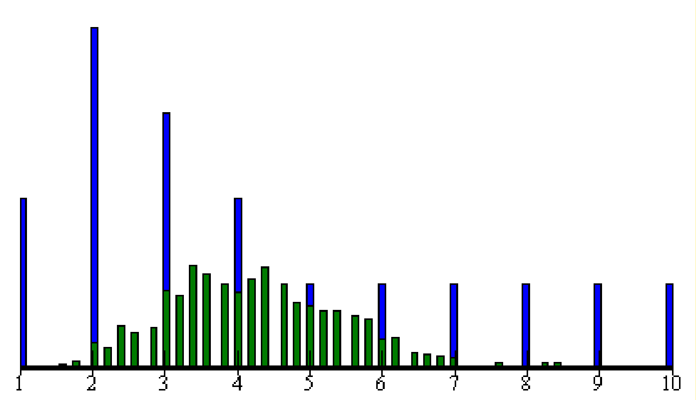
  also called the *standard error* of $\bar{X}$

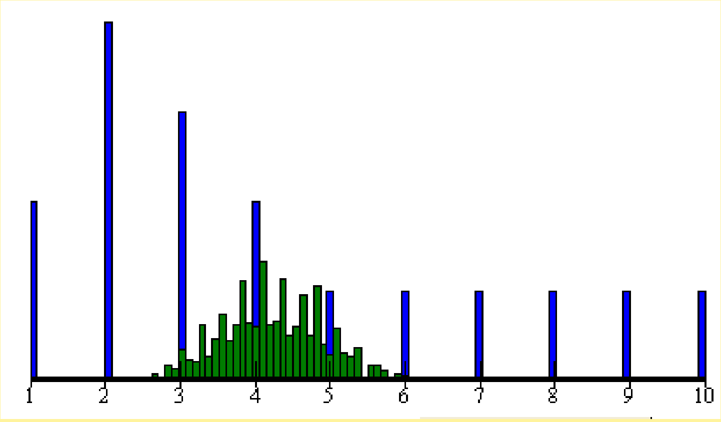
- (Biggie) Central Limit Theorem: As the sample size increases, the distribution of the $\bar{X}$'s gets closer and closer to the normal.

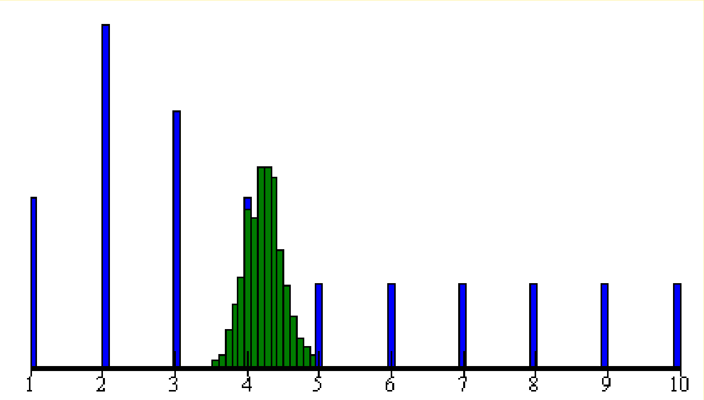
Consequences…

Example of Sampling Distribution of the Mean









As *n* increases, the variability decreases and the normality (bell-shapedness) increases.

Confidence Intervals

- In general, a large sample confidence interval for the mean μ has the form

$$\left[\bar{X} - z\frac{s}{\sqrt{n}}, \bar{X} + z\frac{s}{\sqrt{n}}\right]$$

- Where z is chosen such that the probability under a normal curve within z standard deviations equals the confidence coefficient

Different Confidence Coefficients

- We can use Table B3 to construct confidence intervals for other confidence coefficients
- For example, there is 99% probability that a normal distribution is within 2.575 standard deviations of the mean

  (z = 2.575, tail probability = 0.005)
- A 99% confidence interval for μ *is*

$$\left[\bar{X} - 2.575 \frac{s}{\sqrt{n}}, \bar{X} + 2.575 \frac{s}{\sqrt{n}} \right]$$

Error Probability

- The error probability (α) is the probability that a confidence interval does **not contain the** population parameter
- For a 95% confidence interval, the error probability

  $\alpha = 0.05$
- α = 1 – confidence coefficient, or
- confidence coefficient = 1 – α
- The error probability is the probability that the sample mean $\bar{X}$ falls more than z standard errors from μ (in both directions)
- The confidence interval uses the z-value corresponding to a one-sided tail probability of $\alpha/2$

Different Confidence Coefficients

Confidence Coefficient	α	$\alpha/2$	$z_{\alpha/2}$
.90	.10		
.95			1.96
.98			
.99			2.58
			3.00

Facts about Confidence Intervals

- The width of a confidence interval
  - __________ as the confidence coefficient increases
  - __________ as the error probability decreases
  - __________ as the standard error increases
  - __________ as the sample size increases

Facts about Confidence Intervals II

- If you calculate a 95% confidence interval, say from 10 to 14, there is ***no probability associated with the true unknown parameter*** being in the interval or not
- The true parameter is either in the interval from 10 to 14, or not – we just don’t know it
- The 95% refers to the method: If you repeatedly calculate confidence intervals with the same method, then 95% of them will contain the true parameter

Choice of Sample Size

- So far, we have calculated confidence intervals starting with z, s, n: $\bar{X} \pm z \frac{s}{\sqrt{n}}$
- These three numbers determine the margin of error of the confidence interval: $z \frac{s}{\sqrt{n}}$
- What if we reverse the equation: we specify a desired precision B (bound on the margin of error)???
- Given z and S , we can find the minimal sample size needed for this precision

Choice of Sample Size

- We start with the version of the margin of error that includes the population standard deviation, σ, setting that equal to B:

$$B = z\frac{\sigma}{\sqrt{n}}$$

- We then solve this for n:

$$n = \left\lceil \sigma^2 \left(\frac{z^2}{B^2} \right) \right\rceil$$, where $\lceil\ \rceil$ means “round up”.

Example

- For a random sample of 100 UK employees, the mean distance to work is 3.3 miles and the standard deviation is 2.0 miles.
- Find and interpret a 90% confidence interval for the mean residential distance from work of all UK employees.
- About how large a sample would have been adequate if we needed to estimate the mean to within 0.1, with 90% confidence?

Quiz # 26

- Please write your name and section number on the card:
- What is your plan for thanksgiving?