STA 291
Fall 2009

LECTURE 27
TUESDAY, 1 December

Administrative Notes

- *This week's online homework due on the next Mon.*

- Practice final is posted on the web as well as the old final.
- *Suggested* Reading
  - Study Tools or Textbook Chapter 11

- *Suggested* problems from the textbook:

  11.1 – 11.6

Choice of Sample Size

- We start with the version of the margin of error that includes the population standard deviation, σ, setting that equal to B:

$$B = z\frac{\sigma}{\sqrt{n}}$$

- We then solve this for n:

$$n = \left\lceil \sigma^2 \left(\frac{z^2}{B^2} \right) \right\rceil, \text{ where } \lceil \ \rceil \text{ means “round up”.}$$

Example

- For a random sample of 100 UK employees, the mean distance to work is 3.3 miles and the standard deviation is 2.0 miles.
- Find and interpret a 90% confidence interval for the mean residential distance from work of all UK employees.
- About how large a sample would have been adequate if we needed to estimate the mean to within 0.1, with 90% confidence?

- Fact: it's easier to prove a parameter **isn't** equal to a particular value than it is to prove it **is** equal to a particular value
- Leads to a core notion of hypothesis testing: it's fundamentally a *proof by contradiction*: we set up the belief we wish to disprove as the **null hypothesis** (**H_0**)and the belief we wish to prove as our **alternative hypothesis** (**H_1**) (A.K.A. research hypothesis)

Analogy: Court trial

- In American court trials, jury is instructed to think of the defendant as innocent:

  H_0: Defendant is innocent

- District attorney, police involved, plaintiff, etc., bring every shred evidence to bear, hoping to prove

  H_1: Defendant is guilty

- Which hypothesis is correct?
- Does the jury make the right decision?

Back to statistics …

- Two hypotheses: the null and the alternative
- Process begins with the assumption that the null is *true*
- We calculate a test statistic to determine if there is enough evidence to infer that the alternative is true
- Two possible decisions:
  - Conclude there is enough evidence to reject the null, and therefore accept the alternative.
  - Conclude that there is not enough evidence to reject the null
- Two possible errors?

What about those errors?

Two possible errors:

- Type I error: Rejecting the null when we shouldn't have [P(Type I error) = α]
- Type II error: Not rejecting the null when we should have [P(Type II error) = β]

Hypothesis Testing, example

Suppose that the director of manufacturing at a clothing factory needs to determine whether a new machine is producing a particular type of cloth according to the manufacturer's specifications, which indicate that the cloth should have a mean breaking strength of 70 pounds and a standard deviation of 3.5 pounds. A sample of 49 pieces reveals a sample mean of 69.1 pounds.

"True?" μ

σ

n

$\overline{X}$

Hypothesis Testing, example

Here,

H_0: $\mu = 70$ (what the manufacturer claims)

H_1: $\mu \neq 70$ (our "confrontational" viewpoint)

Other types of alternatives:

H_1: $\mu > 70$

H_1: $\mu < 70$

Hypothesis Testing

- Everything after this—calculation of the test statistic, rejection regions, α, level of significance, p-value, conclusions, etc.—is just a further quantification of the difference between the value of the test statistic and the value from the null hypothesis.

Hypothesis Testing, example

Suppose that the director of manufacturing at a clothing factory needs to determine whether a new machine is producing a particular type of cloth according to the manufacturer's specifications, which indicate that the cloth should have a mean breaking strength of 70 pounds and a standard deviation of 3.5 pounds. A sample of 49 pieces reveals a sample mean of 69.1 pounds. Conduct an $\alpha = .05$ level test.

σ (3.5), $\bar{x}$ (69.1), "True?" μ (70), n (49)

$$\Rightarrow z = \frac{69.1 - 70}{3.5/\sqrt{49}} = -1.80$$

Hypothesis Testing

The *level of significance* is the maximum probability of incorrectly rejecting the null we're willing to accept—a typical value is $\alpha = 0.05$.

The p-*value of a test* is the probability of seeing a value of the test statistic at least as contradictory to the null as that we actually observed, if we assume the null is true.

Hypothesis Testing, example

- Here,

  H_0: $\mu = 70$ (what the manufacturer claims)

  H_1: $\mu \neq 70$ (our "confrontational" viewpoint)

Our test statistic:

$$z = \frac{69.1 - 70}{3.5 / \sqrt{49}} = -1.80$$

Giving a *p*-value of .0359 x 2 = .0718. Because this exceeds the significance level of $\alpha = .05$, we don't reject, deciding there isn't enough evidence to reject the manufacturer's claim

Attendance Question #27

Write your name and section number on your index card.

Today's question: