STA 291
Fall 2009

LECTURE 28
THURSDAY, 3 December

Administrative Notes

- *This week's online homework due on the next Mon.*
- The final will be at CB110 on Tue Dec 15th at 8:30pm to 10:30pm (make-up will be on Wed Dec 16th at 9:30am to 11:30am).
- Practice final is posted on the web as well as the old final.
- *Suggested* Reading
  – Study Tools or Textbook Chapter 11
- *Suggested* problems from the textbook:
  11.1 – 11.6

Review: Significance Tests

- A significance test is used to find evidence ***against*** a hypothesis
- The sampling distribution helps quantify the evidence ("*p*-value")
- Enough evidence against the hypothesis: Reject the hypothesis.
- Not enough evidence: No conclusion.

Elements of a Significance Test

- Assumptions
  - – Type of data, population distribution, sample size
- Hypotheses
  - – Null and alternative hypothesis
- Test Statistic
  - – Compares point estimate to parameter value under the null hypothesis
- P-value
  - – Uses sampling distribution to quantify evidence against null hypothesis
  - – Small P is more contradictory
- Conclusion
  - – Report P-value
  - – Make formal rejection decision (optional)

p-Value

- How unusual is the observed test statistic when the null hypothesis is assumed true?

- The ***p-value*** is the probability, assuming that H_0 is true, that the test statistic takes values at least as contradictory to H_0 as the value actually observed

- The smaller the p-value, the more strongly the data contradict H_0

Decisions and Types of Errors in Tests of Hypotheses

- Terminology:
  - α-level (significance level) is a number such that one rejects the null hypothesis if the *p*-value is less than or equal to it.
  - Often, α=0.05
  - Choice of the α-level reflects how cautious the researcher wants to be ("acceptable risk")
  - Significance level α needs to be chosen ***before*** analyzing the data

Decisions and Types of Errors in Tests of Hypotheses

- More Terminology:

  – The rejection region is a range of values such that if the test statistic falls into that range, we decide to reject the null hypothesis in favor of the alternative hypothesis

Type I and Type II Errors

- Type I Error: The null hypothesis is rejected, even though it is true.

- Type II Error: The null hypothesis is not rejected, even though it is false.

Type I and Type II Errors

		Decision	
		Reject	Do not reject
Condition of the null hypothesis	True	***Type I error***	***Correct***
	False	***Correct***	***Type II error***

Type I and Type II Errors

- Terminology:
  - ***α = Probability of a Type I error***
  - ***β = Probability of a Type II error***
  - ***Power = 1 – Probability of a Type II error***
- The smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- If you ask for very strong evidence to reject the null hypothesis, it is more likely that you fail to detect a real difference

Type I and Type II Errors

- In practice, α is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult
- **How to choose α?**
- If the consequences of a Type I error are very serious, then α should be small.
- For example, you want to find evidence that someone is guilty of a crime
- In exploratory research, often a larger probability of Type I error is acceptable
- If the sample size increases, both error probabilities decrease

Significance Test for a Mean

Example

- The mean score for all high school seniors taking a college entrance exam equals 500.
- A study is conducted to see whether a different mean applies to those students born in a foreign country.
- For a random sample of 100 of such students, the mean and standard deviation on this exam equal 508 and 100.
- Q: Is the mean significantly different from 500 for international students?

Significance Test for a Mean

Assumptions

- What type of data?
  - *Quantitative*
- What is the population distribution?
  - *No special assumptions. The test refers to the population mean of the quantitative variable.*
- Which sampling method has been used?
  - *Random sampling*
- What is the sample size?
  - *Minimum sample size of n=30 to use Central Limit Theorem with estimated standard deviation*

Significance Test for a Mean

Hypotheses

- The null hypothesis has the form H_0: $\mu = \mu_0$, where μ_0 is an *a priori* (before taking the sample) specified number like 0 or 5.3
- The most common alternative hypothesis is

$$H_1: \mu \neq \mu_0$$

- This is called a *two-sided* hypothesis, since it includes values falling above and below the null hypothesis

Significance Test for a Mean

Test Statistic

- The hypothesis is about the population mean
- So, a natural test statistic would be the sample mean
- The sample mean has, for sample size of at least n=25, an approximately normal sampling distribution
- The parameters of the sampling distribution are, under the null hypothesis,

– Mean = μ_0 (that is, the sampling distribution is centered around the hypothesized mean)

– Standard error = $\frac{\sigma}{\sqrt{n}}$, estimated by $\frac{s}{\sqrt{n}}$

Significance Test for a Mean

Test Statistic

- Then, the *z*-score has a standard normal distribution

$$\frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- The *z*-score measures how many estimated standard errors the sample mean falls from the hypothesized population mean
- The farther the sample mean falls from μ_0, the larger the absolute value of the *z* test statistic, and the stronger the evidence against the null hypothesis

p-Value

- The *p*-value has the advantage that different test results from different tests can be compared: The *p*-value is always a number between 0 and 1
- The *p*-value can be obtained from Table Z: It is the probability that a standard normal distribution takes values more extreme than the observed *z* score
- The smaller the *p*-value is, the stronger the evidence against the null hypothesis and in favor of the alternative hypothesis
- Round *p*-value to two or three significant digits

Example

- The mean score for all high school seniors taking a college entrance exam equals 500. A study is conducted to see whether a different mean applies to those students born in a foreign country. For a random sample of 100 of such students, the mean and standard deviation on this exam equal 508 and 100.

1. Set up hypotheses for a significance test.
2. Compute the test statistic.
3. Report the *P*-value, and interpret.
4. Can you conclude that the population mean for students born in a foreign country equals 500? Why or why not?
5. Make a decision about H_0, using α=0.05.

One-Sided Tests of Hypotheses

- Recall: The research hypothesis is usually the alternative hypothesis
- This is the hypothesis that we want to prove by rejecting the null hypothesis
- Assume that we want to prove that is larger than a particular number μ_0
- Then, we need a one-sided test with hypotheses:

$$H_0: \ \mu = \mu_0 \ \text{ vs. } H_1: \ \mu > \mu_0$$

One-Sided Alternative Example

- Example: Usually, Americans eat 2.5 pounds of turkey on Thanksgiving day.
- You want to prove that this year, that figure is too high—that Americans are cutting back.
- You sample $n = 40$ Americans, asking how much they eat.
- Null hypothesis: H_0: $\mu = 2.5$
- Alternative hypothesis: H_1: $\mu < 2.5$

Two-Sided versus One-Sided

- Two-sided tests are more common

- Look for formulations like
  - "test whether the mean has ***changed***"
  - "test whether the mean has ***increased***"
  - "test whether the mean is ***the same***"
  - "test whether the mean has ***decreased***"

Summary

	One-Sided Tests		Two-Sided Test
Null Hypothesis	$H_0 : \mu = \mu_0$		
Research Hypothesis	$H_1 : \mu < \mu_0$	$H_1 : \mu > \mu_0$	$H_1 : \mu \neq \mu_0$
Test Statistic	$z = \dfrac{\bar{X} - \mu_0}{s / \sqrt{n}}$		
p-value	$P(Z < z_{obs})$	$P(Z > z_{obs})$	$2 \cdot P(Z > \lvert z_{obs} \rvert)$

Attendance Survey Question #28

- ***On a 4”x6” index card***
  - Please write down your name and section number
  - Today’s Question: