

STA 291
Spring 2009

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LECTURE 10
THURSDAY, 26 FEBRUARY

Preview of Coming Attractions

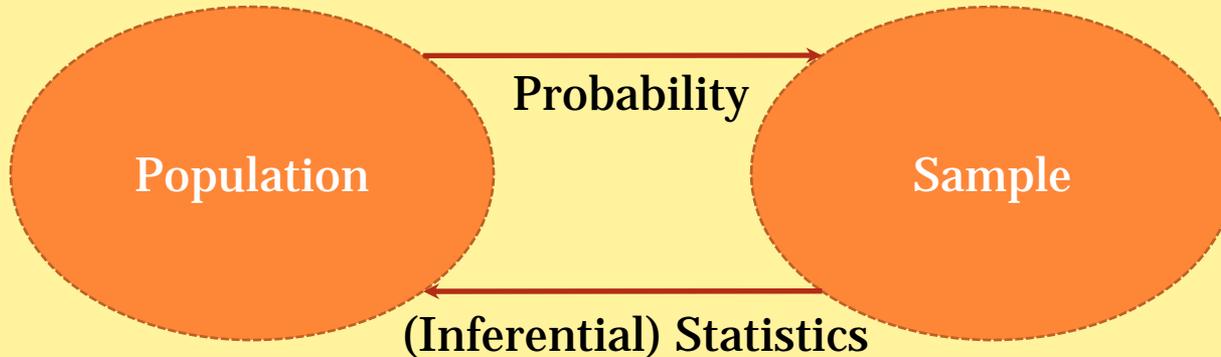
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- **6 Probability**
- **7 Random Variables and Discrete Probability Distributions**

Chapter 6: Probability

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- Abstract but necessary because this is the mathematical theory underlying all statistical inference



- Fundamental concepts that are very important to understanding *Sampling Distribution*, *Confidence Interval*, and *P-Value*
- Our goal for Chapter 6 is to learn the rules involved with assigning probabilities to events

Probability: Basic Terminology

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- **Experiment:** Any activity from which an outcome, measurement, or other such result is obtained.
- **Random (or Chance) Experiment:** An experiment with the property that the outcome cannot be predicted with certainty.
- **Outcome:** Any possible result of an experiment.
- **Sample Space:** The collection of all possible outcomes of an experiment.
- **Event:** A specific collection of outcomes.
- **Simple Event:** An event consisting of exactly one outcome.

Experiments, Outcomes, Sample Spaces, and Events

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Examples:

Experiment

1. Flip a coin
2. Flip a coin 3 times
3. Roll a die
4. Draw a SRS of size 50 from a population

Sample Space

- 1.
- 2.
- 3.
- 4.

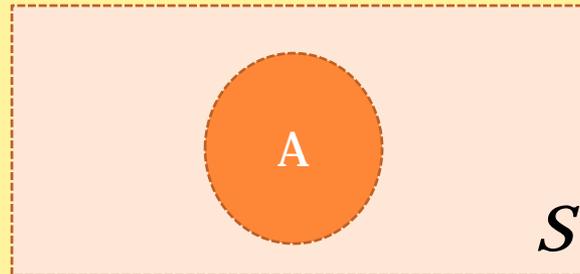
Event

- 1.
- 2.
- 3.
- 4.

Complement

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- Let A denote an event.
- The **complement** of an event A : All the outcomes in the sample space S that do not belong to the event A . The complement of A is denoted by A^c



Law of Complements: $P(A^c) = 1 - P(A)$

Example: If the probability of getting a “working” computer is 0.7, what is the probability of getting a defective computer?

Union and Intersection

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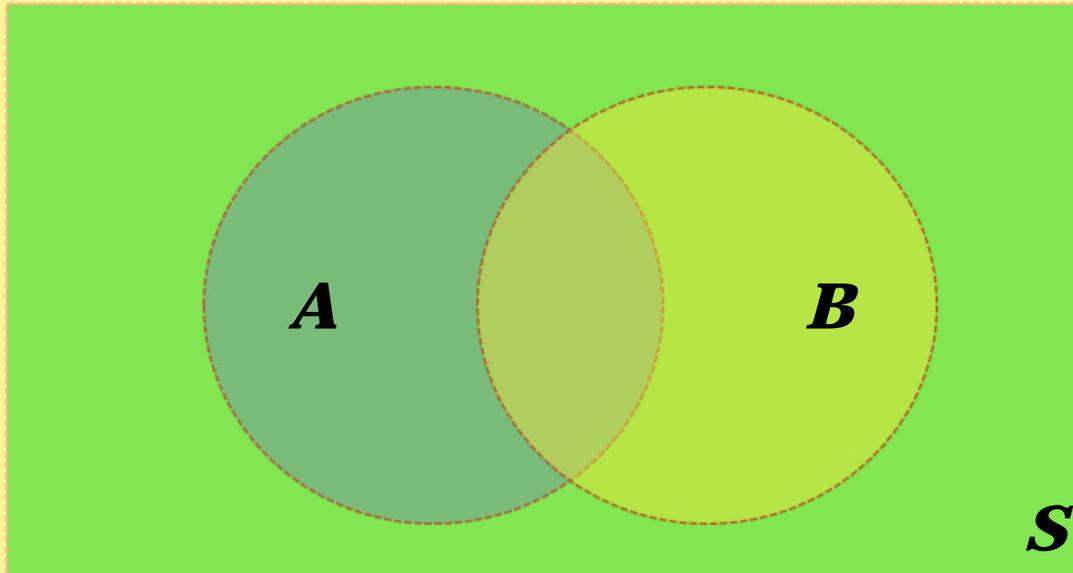
- Let A and B denote two events.
- The **union** of two events: All the outcomes in S that belong to at least one of A or B . The union of A and B is denoted by $A \cup B$
- The **intersection** of two events: All the outcomes in S that belong to both A and B . The intersection of A and B is denoted by $A \cap B$

Additive Law of Probability

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- Let A and B be *any* two events in the sample space S . The probability of the union of A and B is

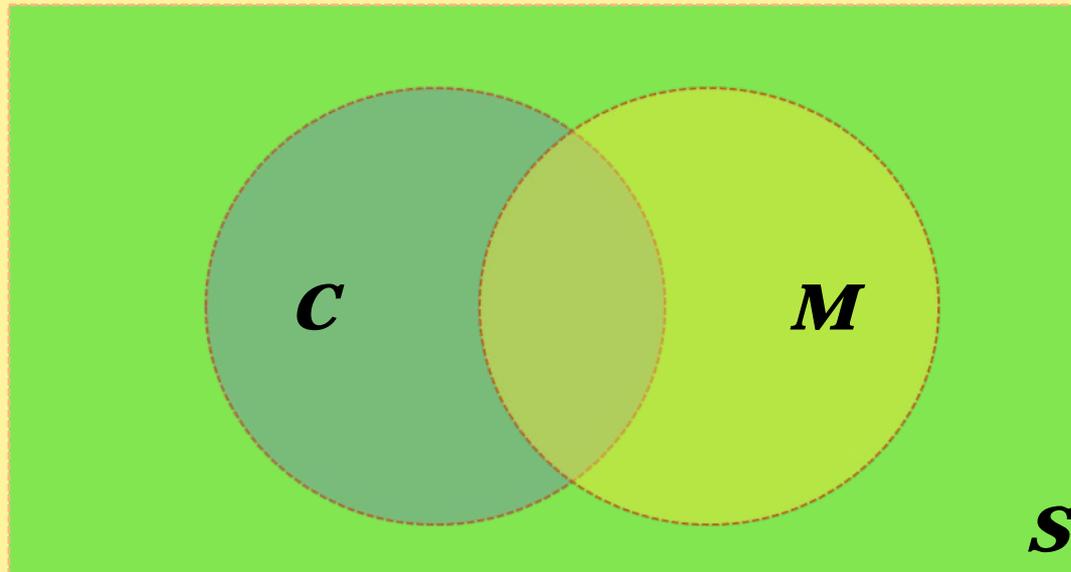
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Additive Law of Probability

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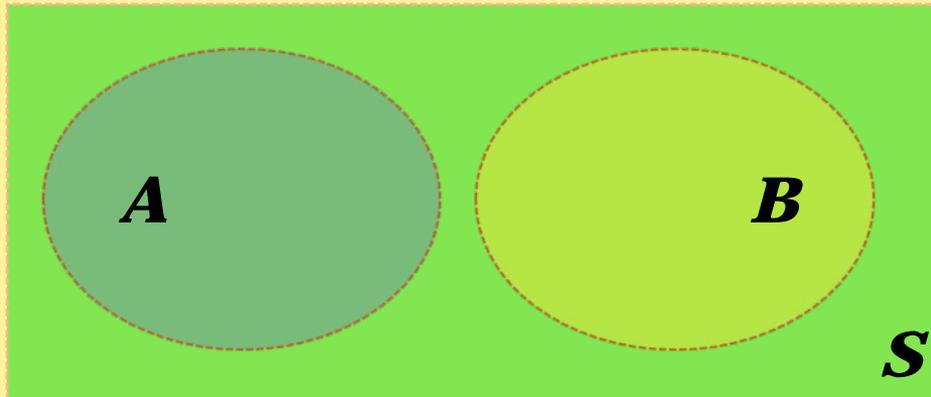
Example: At a large University, all first-year students must take chemistry and math. Suppose 85% pass chemistry, 88% pass math, and 78% pass both. Suppose a first-year student is selected at random. What is the probability that this student passed at least one of the courses?



Disjoint Sets

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- Let A and B denote two events.
- **Disjoint (mutually exclusive) events:** A and B are said to be *disjoint* if there are no outcomes common to both A and B .
- The notation for this is written as $A \cap B = \{ \} = \phi$
- Note: The last symbol denotes the null set or the empty set.



Assigning Probabilities to Events

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- The probability of an event is a value between 0 and 1.
- In particular:
 - 0 implies that the event will never occur
 - 1 implies that the event will always occur
- How do we assign probabilities to events?

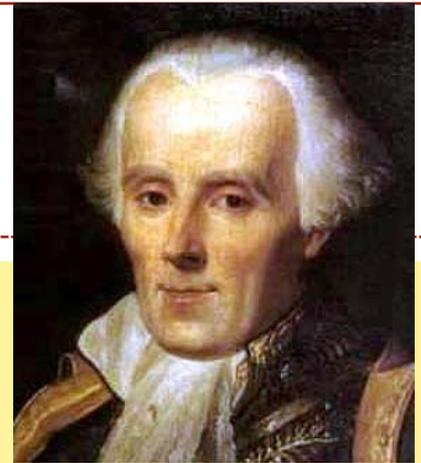
Assigning Probabilities to Events

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- There are different approaches to assigning probabilities to events
- Objective
 - **equally likely outcomes (classical approach)**
 - **relative frequency**
- Subjective

Equally Likely Approach (Laplace)

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- The equally likely outcomes approach usually relies on symmetry/geometry to assign probabilities to events.
- As such, we do not need to conduct experiments to determine the probabilities.
- Suppose that an experiment has only n outcomes. The equally likely approach to probability assigns a probability of $1/n$ to each of the outcomes.
- Further, if an event A is made up of m outcomes, then $P(A) = m/n$.

Equally Likely Approach

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- **Examples:**

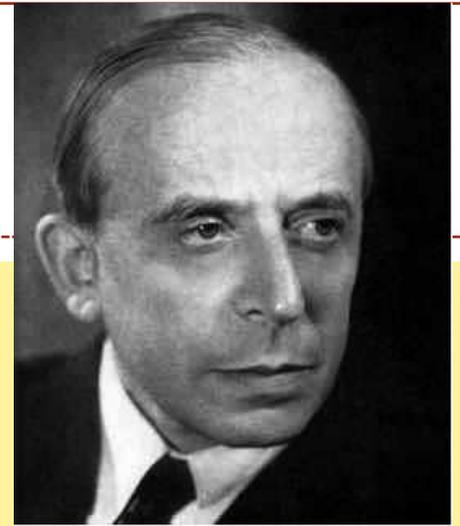
1. **Roll a fair die**

- The probability of getting “5” is $1/6$
- This does not mean that whenever you roll the die 6 times, you definitely get exactly one “5”

2. **Select a SRS of size 2 from a population**

Relative Frequency Approach (von Mises)

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- The relative frequency approach borrows from calculus' concept of limit.
- Here's the process:
 1. Repeat an experiment n times.
 2. Record the number of times an event A occurs. Denote that value by a .
 3. Calculate the value a/n

Relative Frequency Approach

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- We could then define the probability of an event A in the following manner:
- Typically, we can't do the “ n to infinity” in real-life situations, so instead we use a “large” n and say that

$$\text{Prob}(A) = \lim_{n \rightarrow \infty} \frac{a}{n}$$

$$\text{Prob}(A) \approx \frac{a}{n}$$

Relative Frequency Approach

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- What is the formal name of the device that allows us to use “large” n ?
- Law of Large Numbers:
 - As the number of repetitions of a random experiment increases,
 - the chance that the relative frequency of occurrences for an event will differ from the true probability of the event by more than any small number approaches 0.

Subjective Probability

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- A subjective probability relies on a person to make a judgment as to how likely an event will occur.
- The events of interest are usually events that cannot be replicated easily or cannot be modeled with the equally likely outcomes approach.
- As such, these values will most likely vary from person to person.
- The only rule for a subjective probability is that the probability of the event must be a value in the interval $[0,1]$

Probabilities of Events

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Let A be the event $A = \{o_1, o_2, \dots, o_k\}$, where o_1, o_2, \dots, o_k are k different outcomes. Then

$$P(A) = P(o_1) + P(o_2) + \dots + P(o_k)$$

Problem: The number on a license plate is any digit between 0 and 9. What is the probability that the first digit is a 3? What is the probability that the first digit is less than 4?

Conditional Probability & the Multiplication Rule

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$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) \neq 0$$

- Note: $P(A/B)$ is read as “the probability that A occurs given that B has occurred.”
- Multiplied out, this gives *the multiplication rule*:

$$P(A \cap B) = P(B) \times P(A | B)$$

Multiplication Rule Example

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- The multiplication rule:

$$P(A \cap B) = P(B) \times P(A | B)$$

- Ex.: A disease which occurs in .001 of the population is tested using a method with a false-positive rate of .05 and a false-negative rate of .05. If someone is selected and tested at random, what is the probability they are positive, and the method shows it?

Independence

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- If events A and B are independent, then the events A and B have no influence on each other.
- So, the probability of A is unaffected by whether B has occurred.
- Mathematically, if A is independent of B , we write:
$$P(A/B) = P(A)$$

Multiplication Rule and Independent Events

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Multiplication Rule for Independent Events: Let A and B be two independent events, then

$$P(A \cap B) = P(A)P(B).$$

Examples:

- Flip a coin twice. What is the probability of observing two heads?
- Flip a coin twice. What is the probability of getting a head and then a tail? A tail and then a head? One head?
- Three computers are ordered. If the probability of getting a “working” computer is 0.7, what is the probability that all three are “working” ?

Attendance Survey Question 11

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- ***On a your index card:***
 - Please write down your name and section number
 - Today's Question:

What probability reflects an event being *certain* to happen?

Attendance Survey Question 10

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- ***On a your index card:***
 - Please write down your name and section number
 - Today's Question: