STA 291
Spring 2009

LECTURE 12
TUESDAY, 10 MARCH

Homework

- Graded online homework is due Saturday (10/18) – watch for it to be posted today.

- Suggested problems from the textbook:

  7.20, 7.30, 7.84, 7.92, 7.96, 7.106*

Expected Value of a Random Variable

- The Expected Value, or mean, of a random variable, X, is
  Mean = E(X)= $\mu = \sum x_i P(X = x_i)$

- Back to our previous example—what's E(X)?

X	2	4	6	8	10
$P(x)$	.05	.20	.35	.30	.10

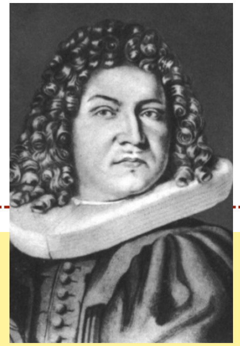
Variance of a Random Variable

- Variance= Var(X) =

$$\sigma^2 = E\left[\left(X-\mu\right)^2\right] = \sum\left(x_i-\mu\right)^2 \cdot P\left(X = x_i\right)$$

- Back to our previous example—what's Var(X)?

X	2	4	6	8	10
$P(x)$	.05	.20	.35	.30	.10



Bernoulli Trial

- Suppose we have a single random experiment X with two outcomes: "success" and "failure."
- Typically, we denote "success" by the value 1 and "failure" by the value 0.
- It is also customary to label the corresponding probabilities as:

$$P(\text{success}) = P(1) = p \text{ and}$$

$$P(\text{failure}) = P(0) = 1 - p = q$$

- Note: $p + q = 1$

Binomial Distribution I

- Suppose we perform several Bernoulli experiments and they are all independent of each other.
- Let's say we do n of them. The value n is the **number of trials.**
- We will label these n Bernoulli random variables in this manner: $X_1, X_2, \ldots, X_n$
- As before, we will assume that the probability of success in a single trial is p, and that this probability of success doesn't change from trial to trial.

Binomial Distribution II

- Now, we will build a new random variable X using all of these Bernoulli random variables:

$$X = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i$$

- What are the possible outcomes of X?
- What is X counting?
- How can we find $P(X = x)$?

Binomial Distribution III

- We need a quick way to count the number of ways in which k successes can occur in n trials.
- Here's the formula to find this value:

$$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}, \text{ where } n! = n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1 \text{ and } 0! \equiv 1$$

- Note: ${}_nC_k$ *is read as* "n choose k."

Binomial Distribution IV

- Now, we can write the formula for the binomial distribution:
- The probability of observing x successes in n independent trials is

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \text{ for } x = 0, 1, \cdots, n$$

under the assumption that the probability of success in a single trial is p.

Using Binomial Probabilities

Note: Unlike generic random variables where we would have to be given the probability distribution or calculate it from a frequency distribution, here we can calculate it from a mathematical formula.

Helpful resources (besides your calculator):

- Excel:

Enter	Gives
=BINOMDIST(4,10,0.2,FALSE)	0.08808
=BINOMDIST(4,10,0.2,TRUE)	0.967207

- Table 1, pp. B-1 to B-5 in the back of your book

Table 1, pp. B-1 to B-5

TABLE 1 Binomial Probabilities

Tabulated values are $P(X \leq k) = \sum_{x=0}^{k} p(xi)$. (Values are rounded to four decimal places.)

$n = 5$

k	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.9510	0.7738	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0010	0.0003	0.0000	0.0000	0.0000
1	0.9990	0.9774	0.9185	0.7373	0.6328	0.5282	0.3370	0.1875	0.0870	0.0308	0.0156	0.0067	0.0005	0.0000	0.0000
2	1.0000	0.9988	0.9914	0.9421	0.8965	0.8369	0.6826	0.5000	0.3174	0.1631	0.1035	0.0579	0.0086	0.0012	0.0000
3	1.0000	1.0000	0.9995	0.9933	0.9844	0.9692	0.9130	0.8125	0.6630	0.4718	0.3672	0.2627	0.0815	0.0226	0.0010
4	1.0000	1.0000	1.0000	0.9997	0.9990	0.9976	0.9898	0.9688	0.9222	0.8319	0.7627	0.6723	0.4095	0.2262	0.0490

$n = 6$

k	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.9415	0.7351	0.5314	0.2621	0.1780	0.1176	0.0467	0.0156	0.0041	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
1	0.9985	0.9672	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.0410	0.0109	0.0046	0.0016	0.0001	0.0000	0.0000
2	1.0000	0.9978	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.0376	0.0170	0.0013	0.0001	0.0000
3	1.0000	0.9999	0.9987	0.9830	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.1694	0.0989	0.0159	0.0022	0.0000
4	1.0000	1.0000	0.9999	0.9984	0.9954	0.9891	0.9590	0.8906	0.7667	0.5798	0.4661	0.3446	0.1143	0.0328	0.0015
5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.8220	0.7379	0.4686	0.2649	0.0585

Binomial Probabilities

We are choosing a random sample of n = 7 Lexington residents—our random variable, C = number of Centerpointe supporters in our sample. Suppose, p = P (Centerpointe support) ≈ 0.3. Find the following probabilities:

a) $P(C = 2)$

b) $P(C < 2)$

c) $P(C \leq 2)$

d) $P(C \geq 2)$

e) $P(1 \leq C \leq 4)$

$n = 7$

k	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.9321	0.6983	0.4783	0.2097	0.1335	0.0824	0.0280	0.0078	0.0016	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.9980	0.9556	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0013	0.0004	0.0000	0.0000	0.0000
2	1.0000	0.9962	0.9743	0.8520	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0129	0.0047	0.0002	0.0000	0.0000
3	1.0000	0.9998	0.9973	0.9667	0.9294	0.8740	0.7102	0.5000	0.2898	0.1260	0.0706	0.0333	0.0027	0.0002	0.0000
4	1.0000	1.0000	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.2436	0.1480	0.0257	0.0038	0.0000
5	1.0000	1.0000	1.0000	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.5551	0.4233	0.1497	0.0444	0.0020
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9984	0.9922	0.9720	0.9176	0.8665	0.7903	0.5217	0.3017	0.0679

What is the *expected* number of Centerpointe supporters, μ_C?

Attendance Question #13

Write your name and section number on your index card.

Today's question: