# STA 291
# Spring 2009



LECTURE 19

THURSDAY, 14 April

# Administrative Notes



- **11 Hypothesis Testing**
  - **11.1 Concepts of Hypothesis Testing**
  - **11.2 Test for the Population Mean**

- *Online homework* ***due this Sat***
- *Suggested* Reading
  - Study Tools or Textbook Chapter 11.2
- *Suggested* problems from the textbook:

  11.7, 11.8, 11.9, 11.13, 11.14, 11.15

# Review: 11.1 Significance Tests



- A significance test is used to find evidence ***against*** a hypothesis

- The sampling distribution helps quantify the evidence ("*p*-value")

- Enough evidence against the hypothesis: Reject the hypothesis.

- Not enough evidence: No conclusion.

# Elements of a Significance Test



- Assumptions
  - Type of data, population distribution, sample size
- Hypotheses
  - Null and alternative hypothesis
- Test Statistic
  - Compares point estimate to parameter value under the null hypothesis
- P-value
  - Uses sampling distribution to quantify evidence against null hypothesis
  - Small P is more contradictory
- Conclusion
  - Report P-value
  - Make formal rejection decision (optional)

# *p*-Value



- How unusual is the observed test statistic when the null hypothesis is assumed true?

- The ***p-value*** is the probability, assuming that $H_0$ is true, that the test statistic takes values at least as contradictory to $H_0$ as the value actually observed

- The smaller the *p*-value, the more strongly the data contradict $H_0$

# Decisions and Types of Errors in Tests of Hypotheses



- Terminology:
  - $\alpha$-level (significance level) is a number such that one rejects the null hypothesis if the *p*-value is less than or equal to it.
  - Often, $\alpha$=0.05
  - Choice of the $\alpha$-level reflects how cautious the researcher wants to be ("acceptable risk")
  - Significance level $\alpha$ needs to be chosen ***before*** analyzing the data

# Decisions and Types of Errors in Tests of Hypotheses



- More Terminology:
  - The rejection region is a range of values such that if the test statistic falls into that range, we decide to reject the null hypothesis in favor of the alternative hypothesis

# Type I and Type II Errors



- Type I Error: The null hypothesis is rejected, even though it is true.

- Type II Error: The null hypothesis is not rejected, even though it is false.

# Type I and Type II Errors



| | | Decision | |
|---|---|---|---|
| | | Reject | Do not reject |
| Condition of the null hypothesis | True | ***Type I error*** | ***Correct*** |
| | False | ***Correct*** | ***Type II error*** |

# Type I and Type II Errors



- Terminology:
  - ***$\alpha$ = Probability of a Type I error***
  - ***$\beta$ = Probability of a Type II error***
  - ***Power = 1 – Probability of a Type II error***
- The smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- If you ask for very strong evidence to reject the null hypothesis, it is more likely that you fail to detect a real difference

# Type I and Type II Errors



- In practice, $\alpha$ is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult
- **How to choose $\alpha$?**
- If the consequences of a Type I error are very serious, then $\alpha$ should be small.
- For example, you want to find evidence that someone is guilty of a crime
- In exploratory research, often a larger probability of Type I error is acceptable
- If the sample size increases, both error probabilities decrease

# 11.2 Significance Test for a Mean



***Example***

- The mean score for all high school seniors taking a college entrance exam equals 500.
- A study is conducted to see whether a different mean applies to those students born in a foreign country.
- For a random sample of 100 of such students, the mean and standard deviation on this exam equal 508 and 100.
- Q: Is the mean significantly different from 500 for international students?

# Significance Test for a Mean



***Assumptions***

- What type of data?
  - *Quantitative*
- What is the population distribution?
  - *No special assumptions. The test refers to the population mean of the quantitative variable.*
- Which sampling method has been used?
  - *Random sampling*
- What is the sample size?
  - *Minimum sample size of n=30 to use Central Limit Theorem with estimated standard deviation*

# Significance Test for a Mean



## ***Hypotheses***

- The null hypothesis has the form $H_0$: $\mu = \mu_0$, where $\mu_0$ is an *a priori* (before taking the sample) specified number like 0 or 5.3
- The most common alternative hypothesis is

$$H_1: \mu \neq \mu_0$$

- This is called a *two-sided* hypothesis, since it includes values falling above and below the null hypothesis

# Significance Test for a Mean



***Test Statistic***

- The hypothesis is about the population mean
- So, a natural test statistic would be the sample mean
- The sample mean has, for sample size of at least $n$=25, an approximately normal sampling distribution
- The parameters of the sampling distribution are, under the null hypothesis,

– Mean = $\mu_0$ (that is, the sampling distribution is centered around the hypothesized mean)

– Standard error = $\frac{\sigma}{\sqrt{n}}$, estimated by $\frac{s}{\sqrt{n}}$

# Significance Test for a Mean



***Test Statistic***

- Then, the $z$-score has a standard normal distribution

$$\frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- The $z$-score measures how many estimated standard errors the sample mean falls from the hypothesized population mean
- The farther the sample mean falls from $\mu_0$, the larger the absolute value of the $z$ test statistic, and the stronger the evidence against the null hypothesis

# *p*-Value



- The *p*-value has the advantage that different test results from different tests can be compared: The *p*-value is always a number between 0 and 1
- The *p*-value can be obtained from Table B3: It is the probability that a standard normal distribution takes values more extreme than the observed *z* score
- The smaller the *p*-value is, the stronger the evidence against the null hypothesis and in favor of the alternative hypothesis
- Round *p*-value to two or three significant digits

# Example



- The mean score for all high school seniors taking a college entrance exam equals 500. A study is conducted to see whether a different mean applies to those students born in a foreign country. For a random sample of 100 of such students, the mean and standard deviation on this exam equal 508 and 100.

1. Set up hypotheses for a significance test.
2. Compute the test statistic.
3. Report the *P*-value, and interpret.
4. Can you conclude that the population mean for students born in a foreign country equals 500? Why or why not?
5. Make a decision about $H_0$, using $\alpha$=0.05.

# One-Sided Tests of Hypotheses



- Recall: The research hypothesis is usually the alternative hypothesis
- This is the hypothesis that we want to prove by rejecting the null hypothesis
- Assume that we want to prove that is larger than a particular number $\mu_0$
- Then, we need a one-sided test with hypotheses:

$$H_0: \ \mu = \mu_0 \ \text{ vs. } H_1: \ \mu > \mu_0$$

# One-Sided Alternative Example



- Example: Usually, Americans eat 2.5 pounds of turkey on Thanksgiving day.
- You want to prove that this year, that figure is too high—that Americans are cutting back.
- You sample $n = 40$ Americans, asking how much they eat.
- Null hypothesis: $H_0$: $\mu = 2.5$
- Alternative hypothesis: $H_1$: $\mu < 2.5$

# Two-Sided versus One-Sided



- Two-sided tests are more common

- Look for formulations like
  - "test whether the mean has ***changed***"
  - "test whether the mean has ***increased***"
  - "test whether the mean is ***the same***"
  - "test whether the mean has ***decreased***"

# Summary



| | One-Sided Tests | | Two-Sided Test |
|---|---|---|---|
| Null Hypothesis | $H_0 : \mu = \mu_0$ | | |
| Research Hypothesis | $H_1 : \mu < \mu_0$ | $H_1 : \mu > \mu_0$ | $H_1 : \mu \neq \mu_0$ |
| Test Statistic | $z = \dfrac{\bar{X} - \mu_0}{s / \sqrt{n}}$ | | |
| *p*-value | $P(Z < z_{obs})$ | $P(Z > z_{obs})$ | $2 \cdot P(Z > \lvert z_{obs} \rvert)$ |

# Attendance Survey Question #19



- ***On a 4”x6” index card***
  - Please write down your name and section number
  - Today’s Question: