STA 291
Spring 2009

LECTURE 21
THURS, 23 April

Administrative Notes

- **12 Hypothesis Testing**
  - **12.1 Small Sample Inference about a Population Mean**
- **13 Comparing Two Populations**
  - **13.1 Comparison of Two Groups: Independent Samples**
- *Last online homework! HW 12, due Sat, 11pm*
- *Suggested* Reading
  - Sections 12.1 and 12.3 in the textbook/study tools
- *Suggested* problems from the textbook:

  12.2, 12.8, 12.12, 12.57, 12.70

12.1 Small Sample Confidence Interval for a Mean

- What if we want to make inference about the population mean, but our sample size is not big enough to meet the minimal sample size requirement n>25 to apply the Central Limit Theorem?
- Confidence intervals are constructed in the same way as before, but now we are using ***t-values*** instead of ***z-values***

12.1 Small Sample Confidence Interval for a Mean

- For a random sample ***from a normal distribution***, a 95% confidence interval for μ is

$$\bar{x} \pm t_{0.025} \frac{s}{\sqrt{n}}$$

- where $t_{0.025}$ is a *t*-score (instead of *z*-score) from Table B4 (p. B-9) or better, from a site like *surfstat:*
- http://www.anu.edu.au/nceph/surfstat/surfstat-home/tables/t.php
- degrees of freedom are $df = n - 1$

Small Sample Hypothesis Test for a Mean

- Assumptions
  - Quantitative variable, random sampling, **population distribution is normal, *any sample size***
- Hypotheses
  - Same as in the large sample test for the mean

$$H_0: \quad \mu = \mu_0$$

$$H_1: \quad \text{one of} \quad \mu \neq \mu_0, \quad \mu > \mu_0, \quad \mu < \mu_0$$

Small Sample Hypothesis Test for a Mean

- Test statistic
  - Exactly the same as for the large sample test

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

- *p* - Value
  - Same as for the large sample test (one-or two-sided), but using the table/online tool for the *t* distribution
  - Table B4 only provides very few values
- Conclusion
  - Report *p*-value and make formal decision

Example

- A study was conducted of the effects of a special class designed to improve children/s verbal skills
- Each child took a verbal skills test twice, both before and after a three-week period in the class
- X=2nd exam score – 1st exam score
- If the population mean for X, $E(X)=\mu$ equals 0, the class has no effect
- Test the null hypothesis of no effect against the alternative hypothesis that the effect is positive
- Sample (n = 4): 3, 7, 3 , 3

Normality Assumption

- An assumption for the *t*-test is that the population distribution is normal
- In practice, it is impossible to be 100% sure if the population distribution is normal
- It is useful to look at histogram or stem-and- leaf plot (or normal probability plot) to check whether the normality assumption is reasonable

Normality Assumption

- Good news: The *t*-test is relatively **robust** against violations of the assumption that the population distribution is normal
- Unless the population distribution is highly skewed, the *p*-values und confidence intervals are fairly accurate
- However: The random sampling assumption must never be violated, otherwise the test results are completely invalid

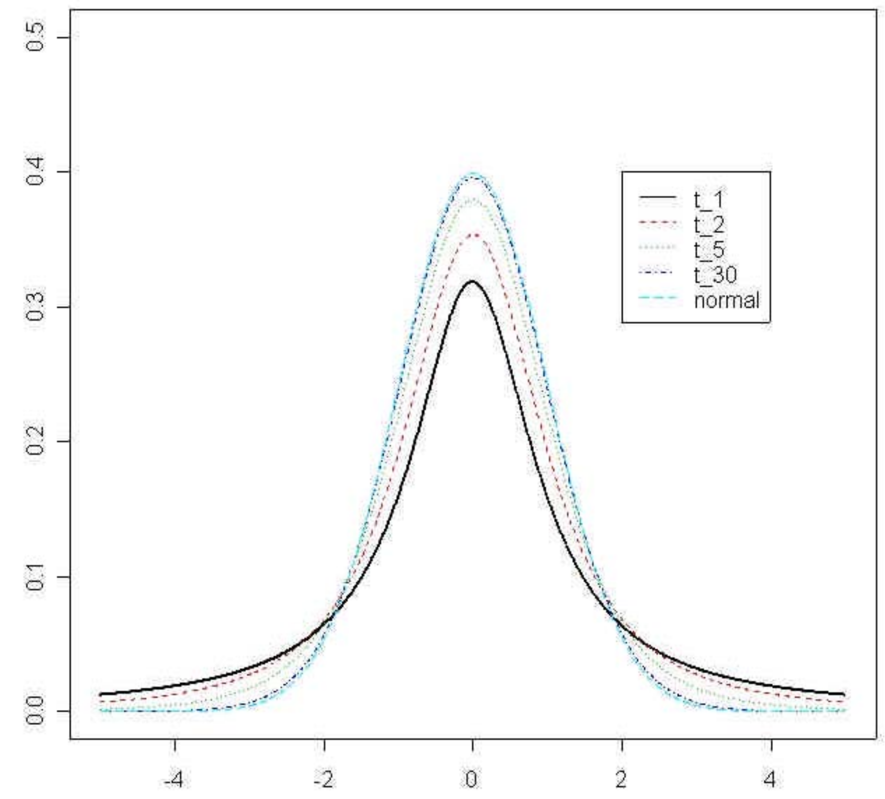
Summary: Small Sample… Significance Test for a Mean

(Assumption: Population distribution is normal)

	One-Sided Tests		Two-Sided Test
Null Hypothesis	$H_0 : \mu = \mu_0$		
Research Hypothesis	$H_1 : \mu < \mu_0$	$H_1 : \mu > \mu_0$	$H_1 : \mu \neq \mu_0$
Test Statistic	$t_{obs} = \dfrac{\bar{X} - \mu_0}{s / \sqrt{n}}$, degrees of freedom $= n - 1$		
p-value	$P(T_{n-1} < t_{obs})$	$P(T_{n-1} > t_{obs})$	$2 \cdot P(T_{n-1} > \lvert t_{obs} \rvert)$

t-Distributions (Section 8.4)

- The t-distributions are bell-shaped and symmetric around 0
- The smaller the degrees of freedom, the more spread out is the distribution
- t-distributions look almost like a normal distribution
- In fact, the limit of the t-distributions is a normal distribution when n gets larger



Summary I

- Testing the Mean
  - Large sample size (*30 or more):*

    Use the large sample test for the mean

    (Table B3, normal distribution)
  - Small sample size:

    Check to be sure the data are not very skewed

    Use the *t*-test for the mean

    (Table B4, *t*-distribution)

Summary II

- Testing the Proportion
  - Large sample size ($np > 5$, $n(1 - p) > 5$):

    Use the large sample test for the proportion

    (Table B3, normal distribution)
  - Small sample size:

    Binomial distribution

13.1 Comparison of **Two Groups** Independent Samples

- Two ***Independent*** Samples
  - Different subjects in the different samples
  - Two subpopulations (e.g., male/female)
  - The two samples constitute independent samples from two subpopulations
  - For example, stratified samples

Comparison of Two Groups
Dependent Samples

- Two ***Dependent*** Samples
  - Natural matching between an observation in one sample and an observation in the other sample
  - For example, two measurements at the same subject (left and right hand, performance before and after training)

- Data sets with dependent samples require different statistical methods than data sets with independent samples

Comparing Two Means (Large Samples)

- Response variable: Quantitative

- Inference about the population means for the two groups, and their difference

$$\mu_1 - \mu_2$$

- Confidence interval for the difference

- Significance test about the difference

Confidence Interval for the Difference of Two Means

- The large sample (both samples sizes at least 20) confidence interval for $\mu_1 - \mu_2$ is

$$\bar{x}_1 - \bar{x}_2 \pm z\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Confidence Interval for the Difference of Two Means: Example

- In a 1994 survey, 350 subjects reported the amount of turkey consumed on Thanksgiving day. The sample mean was 3.1 pounds, with standard deviation 2.3 pounds
- In a 2006 survey, 1965 subjects reported an average amount of consumed Thanksgiving turkey of 2.8 pounds, with standard deviation 2.0 pounds
- *Construct a 95% confidence interval for the difference between the means in 1994 and 2006.*
- *Is it plausible that the population mean was the same in both years?*

Attendance Survey Question #21

- ***On a 4”x6” index card***
  - Please write down your name and section number
  - Today’s Question:

Multiple choice: When using (Gosset’s) *t*-distribution, we have to assume the __________ is normal.

a) sample
b) sampling distribution
c) population
d) parameter