

Name\_\_\_\_\_

## STA 320 Midterm 2

Probability

November 21st, 2012

There are five questions on this test. DO use calculators if you need them. “And then a miracle occurs” is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 75 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

**Question 1: Joint density function**

The joint probability function of two discrete random variables  $X$  and  $Y$  is given by  $f(x, y) = c(2x + y)$ , where  $x$  and  $y$  are integers such that  $0 \leq x \leq 2$ , and  $0 \leq y \leq 3$ , and  $f(x, y) = 0$  otherwise.

(a) Find the value of the constant  $c$ .

(b) Find  $P(X = 2, Y = 1)$ .

(c) Find  $P(X \geq 1, Y \leq 2)$ .

(d) Are  $X$  and  $Y$  independent or dependent? Please show the proof.

**Question 2: Moment generating functions**

A random variable  $X$  takes values  $-1$  and  $1$  with probability  $1/2$  each.

(a) Find the moment generating function.

(b) Compute the first, second, and third moments of  $X$ .

**Question 3: Expectation, Variance, Covariance and Correlation**

The joint probability function of two discrete random variables  $X$  and  $Y$  is given by  $f(x, y) = c(2x + y)$ , where  $x$  and  $y$  are integers such that  $0 \leq x \leq 2$ , and  $0 \leq y \leq 3$ , and  $f(x, y) = 0$  otherwise. The constant  $c$  is the same value in Question 1 (a).

Compute  $\mathbb{E}(X)$ ,  $\mathbb{E}(Y)$ ,  $\text{Var}(X)$ ,  $\text{Var}(Y)$ , and  $\rho$  which is the correlation of  $X$  and  $Y$ .

**Question 4: Discrete Random Variables**

Consider the Binomial random variable  $X$  with the number of trials  $n$  and the probability  $p$ .

(a) Find the moment generating function.

(b) Using the moment generating function compute  $\mathbb{E}(X)$ .

**Question 5: Words problem**

Under average driving conditions, the life lengths of automobile tires of a certain brand are found to follow an exponential distribution, with a mean of 30,000 miles. Find the probability that one of these tires, bought today, will last the following numbers of miles.

(a) Over 30,000 miles

(b) Over 30,000 miles given that it already has gone 15,000 miles.

(c) At a temperature of  $25^{\circ}C$ , the resistance of a type of thermistor are normally distributed with a mean of 10,000 ohm and a standard deviation of 4,000 ohm. The thermistors are to be sorted and those having resistance between 8,000 and 15,000 ohms are to be shipped to a vendor. What fraction of these thermistors will actually be shipped?