HOMEWORK 5

STA320.01, Probability
Fall Semester, 2013

Due: Thurs Oct 24th 2013

1 Consider the following bivariate distribution:

	$P(X,Y)$	Y = 5	10	15
	1	1/9	1/9	0
X	2	1/6	2/9	1/6
	3	0	0	2/9

(a) Show that $P(X,Y)$ is a valid probability distribution function.
(b) Show that X and Y are not independent.
(c) Find the marginal distribution of X.
(d) Find $P(X = 1|Y < 15)$.

2 Consider the folloing joint cumulative distribution function of X and Y:

$$F(x,y) = \begin{cases} x^2y^2, & 0 < x, y < 1 \\ 1, & x, y \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $P(0 < X \leq 0:5, 0:25 \leq Y \leq 0:5)$.
(b) Find $P(0 < Y < 0.5)$.
(c) Find $P(X = Y)$.
(d) Find $P(0 < Y < 0.5, X < 0.2)$.

3 Suppose the cumulative distribution function of X and Y is given by:

$$F(x,y) = \begin{cases} \frac{1}{2}\left[\frac{x^2y}{2} + \frac{3y^2x}{2}\right], & 0 < x, y < 1 \\ 1, & x, y \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $P(0 < X < 1/2, 0 < Y < 1/2)$.
(b) If the marginal cumulative distribution functions of X and Y are

$$G(x) = \begin{cases} \frac{1}{2}\left[\frac{x^2}{2} + \frac{3x}{2}\right], & 0 < x < 1 \\ 1, & x \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

and

$$H(y) = \begin{cases} \frac{1}{2}\left[\frac{y}{2} + \frac{3y^2}{2}\right], & 0 < y < 1 \\ 1, & y \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

respectively.
(i) Find the probability that $X < 1/2$ given $Y \geq 1/2$.
(ii) Find $P(Y < 3/4|Y \geq 1/2)$.

4 Suppose the joint cumulative distribution function of two random variables X and Y is given by

$$F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y}, & x, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 2, Y < 2)$.
(b) Find $P(X < 5)$.
(c) Find $P(1 < X < 3, 2 < Y < 4)$.

5 In a large multinational company, the following is the joint probability distribution function between an employee's gender (X) and whether the person holds an executive position Y ($= 1$ executive position; $= 0$ non-executive position):

		Y	
	$P(X,Y)$	0	1
X	F	0.49	0.01
	M	0.41	0.09

Determine whetether X and Y are independent and use your knowledge about conditional probability and independence to decide whether there is sexual bias in promotion to an excutive position within the company.