

Solution - HW 5

i) $0 < P(X=x, Y=y) < 1 \quad \forall x, y$ ii) $\sum_{x \in X} \sum_{y \in Y} P(X=x, Y=y) = 1$

more specifically,
 $\frac{1}{9} + \frac{1}{9} + 0 + \frac{1}{6} + \frac{2}{9} + \frac{1}{6} + \frac{2}{9}$
 $\rightarrow 1 = 1$

b) $P(X=1, Y=5) = \frac{1}{9}$ $P(X=1)P(Y=5) = \frac{5}{81}$

$P(X=1) = \frac{2}{9}$
 $P(Y=5) = \frac{5}{18}$ So $P(X=1, Y=5) \neq P(X=1)P(Y=5)$

c) $P(X=x) = \sum_{y \in Y} P(X=x, Y=y)$
 $P(X=1) = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$
 $P(X=2) = \frac{1}{6} + \frac{2}{9} + \frac{1}{6} = \frac{5}{9}$
 $P(X=3) = \frac{2}{9}$

d) $P(X=1 | Y < 5) = \frac{P(X=1, Y < 5)}{P(Y < 5)}$
 $= \frac{P(X=1, Y=5) + P(X=1, Y=10)}{P(Y=5) + P(Y=10)}$
 $= \frac{\frac{1}{9} + \frac{1}{9}}{(\frac{1}{9} + \frac{1}{6}) + (\frac{1}{9} + \frac{2}{9})}$
 $= \frac{4}{11}$

e) a) $P(0 < X \leq 0.5, 0.25 \leq Y \leq 0.5)$
 $= P(0 < X \leq 0.5, 0 \leq Y \leq 0.5) - P(0 < X \leq 0.5, 0 \leq Y \leq 0.25)$
 $= F(0.5, 0.5) - F(0.5, 0.25)$
 $= (0.5)^2(0.5)^2 - (0.5)^2(0.25)^2$
 $= 0.04688$

b) $P(0 < Y < 0.5) = P(0 < X < 1, 0 < Y < 0.5)$
 $= 1^2(0.5)^2$
 $= 0.25$

2c) $P(X=Y) = 0$ b/c X and Y are continuous.

$$\begin{aligned} \text{d) } P(0 < Y < 0.5 | X < 0.2) &= \frac{P(X < 0.2, 0 < Y < 0.5)}{P(X < 0.2, 0 < Y < 1)} \\ &= \frac{F(0.2, 0.5)}{F(0.2, 1)} \\ &= \frac{(0.2)^2 (0.5)^2}{(0.2)^2 \cdot 1^2} = 0.25 \end{aligned}$$

$$\begin{aligned} \text{3a) } P(0 < X < 0.5, 0 < Y < 0.5) &= F(0.5, 0.5) \\ &= 0.5 \left(\frac{0.5^2 \cdot 0.5}{2} + \frac{3 \cdot 0.5^2 \cdot 0.5}{2} \right) \\ &= 0.125 \end{aligned}$$

$$\begin{aligned} \text{b) i) } P(X < 1/2 | Y \geq 1/2) &= \frac{P(X < 1/2, Y \geq 1/2)}{P(Y \geq 1/2)} \\ &= \frac{P(X < 1/2) - P(X < 1/2, Y < 1/2)}{1 - P(Y < 1/2)} \\ &= \frac{G(1/2) - F(1/2, 1/2)}{1 - H(1/2)} \\ &= \frac{7/16 - 1/8}{1 - 5/16} \\ &= 0.4545 \end{aligned}$$

$$\begin{aligned}
36 \text{ ii) } P(Y < 3/4 \mid Y \geq 1/2) &= \frac{P(Y < 3/4, Y \geq 1/2)}{P(Y \geq 1/2)} \\
&= \frac{P(Y < 3/4) - P(Y < 1/2)}{1 - P(Y < 1/2)} \\
&= \frac{H(3/4) - H(1/2)}{1 - H(1/2)} \\
&= \frac{\frac{39}{64} - \frac{5}{16}}{1 - \frac{5}{16}} = 0.4318
\end{aligned}$$

$$\begin{aligned}
4a) P(X < 2, Y < 2) &= F(2, 2) \\
&= 1 - e^{-2} - e^{-2} + e^{-2-2} \\
&= 0.7476
\end{aligned}$$

$$\begin{aligned}
b) P(X < 5) &= P(X < 5, Y < \infty) = F(5, \infty) \\
&= 1 - e^{-x} - e^{-y} + e^{-x-y} \\
&= 1 - e^{-x} \\
&= 0.9933
\end{aligned}$$

$$\begin{aligned}
c) P(1 < X < 3, 2 < Y < 4) &= P(X < 3, Y < 4) - P(X < 1, Y < 4) - P(X < 3, Y < 2) \\
&\quad + P(X < 1, Y < 2) \\
&= F(3, 4) - F(1, 4) - F(3, 2) + F(1, 2) \\
&= 0.0372
\end{aligned}$$

$$\begin{aligned}
5) P(X=5, Y=0) &= 0.49 \\
P(X=5)P(Y=0) &= 0.5 \cdot 0.9 = 0.45 \quad \text{so } P(X, Y) \neq P(X)P(Y)
\end{aligned}$$

Evidence for bias, but there could be confounding variables and no bias.