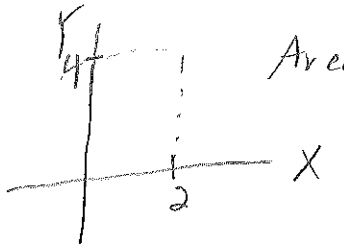
HW 7 Solution

1a)



Area = 8

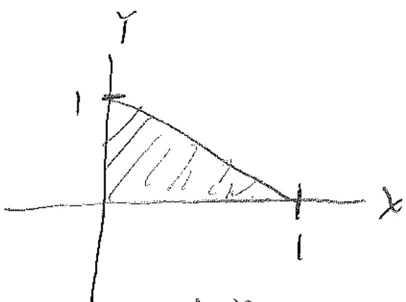
$f_{X,Y}(x,y) = \frac{1}{8}$ since $\int_x \int_y p(x,y)$ must $= 1$
when $0 \le x \le 2,\ 0 \le y \le 4$

$f_X(x) = \int_0^4 \frac{1}{8}\, dy = \frac{1}{8}(4-0) = \frac{1}{2}$

$f_Y(y) = \int_0^2 \frac{1}{8}\, dx = \frac{1}{8}(2-0) = \frac{1}{4}$

b) Yes. $P(X=x, Y=y) = P(X=x)\, P(Y=y)$

2)



$f_Y(y) = \int_0^{1-y} 24xy\, dx = 24x \int_0^{1-y} y\, dy$

$= 24x\left[\frac{1}{2}(1-x)^2\right]$

$= 12x(1-x)^2$

$f_X(x) = \int_0^{1-x} 24xy\, dx = 12y(1-y)^2$

$f_X(x)\, f_Y(y) = 12y(1-y)^2\, 12x(1-x)^2$
$= 144xy(1-y)^2(1-x)^2 \neq 24xy = f(x,y)$

∴ not independent.

3 $P(3X>Y \mid 1<4Z<2) = \dfrac{P(3X>Y,\ 1<4Z<2)}{P(1<4Z<2)} = \dfrac{1/6}{1/4}$ $\boxed{= \frac{2}{3}}$

$P(3X>Y, 1<4Z<2) = \int_{1/4}^{1/2}\int_0^1\int_{Y/3}^{Y} 2\, dx\, dy\, dZ$

$P(3X>Y, \frac{1}{4}<Z<\frac{1}{2}) =$

$= \int_{1/4}^{1/2}\int_0^1 [2x]_{Y/3}^{Y}\, dy\, dZ$

$= \int_{1/4}^{1/2}\int_0^1 (2y - \frac{2}{3}Y)\, dy\, dZ$

$= \int_{1/4}^{1/2} \left[Y^2 - \frac{Y^3}{3}\right]_0^1 dZ$

$= \int_{1/4}^{1/2} \frac{2}{3}\, dZ$

$= \frac{2}{3} Z \Big|_{1/4}^{1/2}$

$= 1/6$

$f(z) \quad = \int_0^1\int_x^1 2\, dy\, dx = \int_0^1 2y\Big|_x^1\, dx$

$= \int_0^1 (2-2x)\, dx$

$= (2x - x^2)\Big|_0^1$

$= 1 \qquad 0<Z<1$

CDF PDF

$F(z) = \int_0^z f(z) = z$

$P(\frac{1}{4}<Z<\frac{1}{2}) = F(1/2) - F(1/4)$

$= 1/2 - 1/4$

$= \frac{1}{4}$

for cont. uniform

4. $Var[X] = \frac{(b-a)^2}{12} = \frac{1}{12}$

or

$E[X] = \int_0^1 x\,dx = \frac{1}{2}x|_0^1 = 1/2$

$E[X^2] = \int_0^1 x^2\,dx = \frac{1}{3}x^3|_0^1 = \frac{1}{3}$

$Var[X] = E[X^2] - E[X]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

5. $E[X(X-1)] = E[X^2 - X]$

$= E[X^2] - E[X]$

$= Var[X] + E[X]^2 + E[X]$

$= \mu^2 + \mu + \sigma^2$

$= \mu(\mu + 1) + \sigma^2$