

Name_____

STA 320 Practice Midterm 2

Probability

November 21st, 2012

There are five questions on this test. DO use calculators if you need them. “And then a miracle occurs” is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 75 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: Joint density function

The joint probability density function of two continuous random variables X and Y is

$$f(x, y) = \begin{cases} cxy & \text{if } 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c .

(b) Find $P(X \geq 3, Y \leq 2)$.

(c) Find $P(1 < X < 2, 2 < Y < 3)$.

(d) Find $P(X + Y < 3)$.

Question 2: Moment generating functions

A random variable X has the probability density function given by

$$f(x) = \begin{cases} 2e^{-2x} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the moment generating function.

(b) Compute the first, second, and third moments of X .

Question 3: Expectation, Variance, Covariance and Correlation

From Question 1. The joint probability density function of two continuous random variables X and Y is

$$f(x, y) = \begin{cases} cxy & \text{if } 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise,} \end{cases}$$

where the constant c is the same value in Question 1 (a).

Compute $\mathbb{E}(X)$, $\mathbb{E}(Y)$, $\text{Var}(X)$, $\text{Var}(Y)$, and ρ which is the correlation of X and Y .

Question 4: Discrete Random Variables

Consider the Poisson random variable X with the mean $\lambda > 0$.

(a) Find the moment generating function.

(b) Using the moment generating function compute $\mathbb{E}(X)$, $\text{Var}(X)$.

Question 5: Words problem

The number of defective circuit boards among those coming out of a soldering machine follows a Poisson distribution. For a particular 8-hour day, one defective board is found.

(a) Find the probability that it was produced during the first hour of operation for that day.

(b) Find the probability that it was produced during the last hour of operation for that day.

(c) The weekly amount spent for maintenance and repair in a certain company has an approximately normal distribution with a mean of \$400 and a standard deviation of \$20. If \$450 is budget to cover repairs for next week, what is the probability that the actual costs will exceed the budget amount?