

HW 1 Solution

1a) $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $p(x_i) = \frac{1}{10}$ $i = 1, \dots, 10$

b) No for 2 reasons. 1) The sample space will differ since an area code cannot start with 0 or 1 and area codes starting with other numbers may not be in a local phone book. 2) Each number in the sample space will not be equally likely.

2)a. Differ by 0: $A = \{\{1, 1\}, \{2, 2\}, \{3, 3\}, \{4, 4\}, \{5, 5\}, \{6, 6\}\}$

Differ by 1: $B = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}, \{2, 1\}, \{3, 2\}, \{4, 3\}, \{5, 4\}, \{6, 5\}\}$

Let S be the sample space. $|S| = 6^2 = 36$
 $|A| = 6$, $|B| = 10$

$$P(A \cup B) = \frac{|A| + |B|}{|S|} = \frac{16}{36}$$

b. $A = \{\{1, 5\}, \{2, 5\}, \{3, 5\}, \{4, 5\}, \{5, 5\}, \{5, 6\}, \{5, 1\}, \{5, 2\}, \{5, 3\}, \{5, 4\}, \{6, 5\}, \{1, 6\}, \{2, 6\}, \{3, 6\}, \{4, 6\}, \{6, 6\}, \{6, 1\}, \{6, 2\}, \{6, 3\}, \{6, 4\}\}$

Let S be the sample space.

$$P(A) = \frac{|A|}{|S|} = \frac{20}{36}$$

A = set of all events with 2 Heads

$$3. A = \left\{ \begin{aligned} &\{HHHTT\}, \{HTHTT\}, \{HHTHT\}, \{HTTHT\}, \{THTHT\}, \\ &\{THTHT\}, \{THTHT\}, \{THTHT\}, \{THTHT\}, \\ &\{THTHT\} \end{aligned} \right\}$$

$$|A| = 10 \quad \text{or} \quad |A| = \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{120}{2 \cdot 6} = 10$$

B = set of all events with 4 heads.

$$B = \left\{ \begin{aligned} &\{HHHHT\}, \{HHTHH\}, \{HHTHH\}, \{HHTHH\}, \\ &\{THTHH\} \end{aligned} \right\}$$

$$|B| = 5 \quad \text{or} \quad |B| = \binom{5}{4} = \frac{5!}{4!(5-4)!} = \frac{120}{24} = 5$$

C = set of all events with 0 heads.

$$C = \left\{ \{TTTTT\} \right\} \quad |C| = 1$$

S = sample space $|S| = 2^5 = 32$

D = set of all events with even # Heads

$$P_r[D] = \frac{|A| + |B| + |C|}{|S|} = \frac{10 + 5 + 1}{32} = \frac{16}{32} = 0.5$$

4. 170 men, so $320 - 170 = 150$ women.

100 men, 160 total go to college so $160 - 100 = 60$

women go to college. $150 - 60 = 90$ women do not go to college

$$6. P(A^c \cap B^c) = P(A \cup B)^c \quad \text{De Morgan's Law} \\ = 1 - P(A \cup B) \\ = 1 - 0.3 - 0.5 \quad \text{disjoint events} \\ = 0.2$$

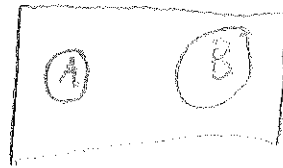
6. alt. $P(A^c) = 1 - P(A) = 0.7$
 $P(B^c) = 1 - P(B) = 0.5$

$$P(A^c \cap B^c) = P(A^c) + P(B^c) - P(A^c \cup B^c)$$

$$= 0.7 + 0.5 - 1$$

$$= 0.2$$

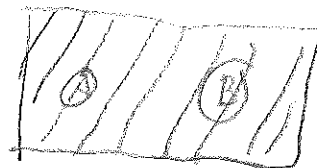
not disjoint



Disjoint



B^c



A^c



$A^c \cup B^c = 1$