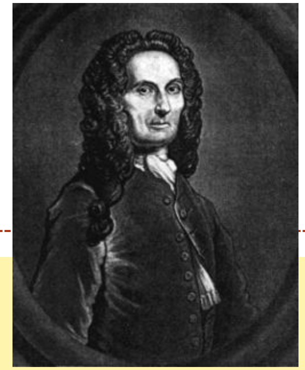
STA 320
Fall 2013

November 14th 2013

The Normal Distribution

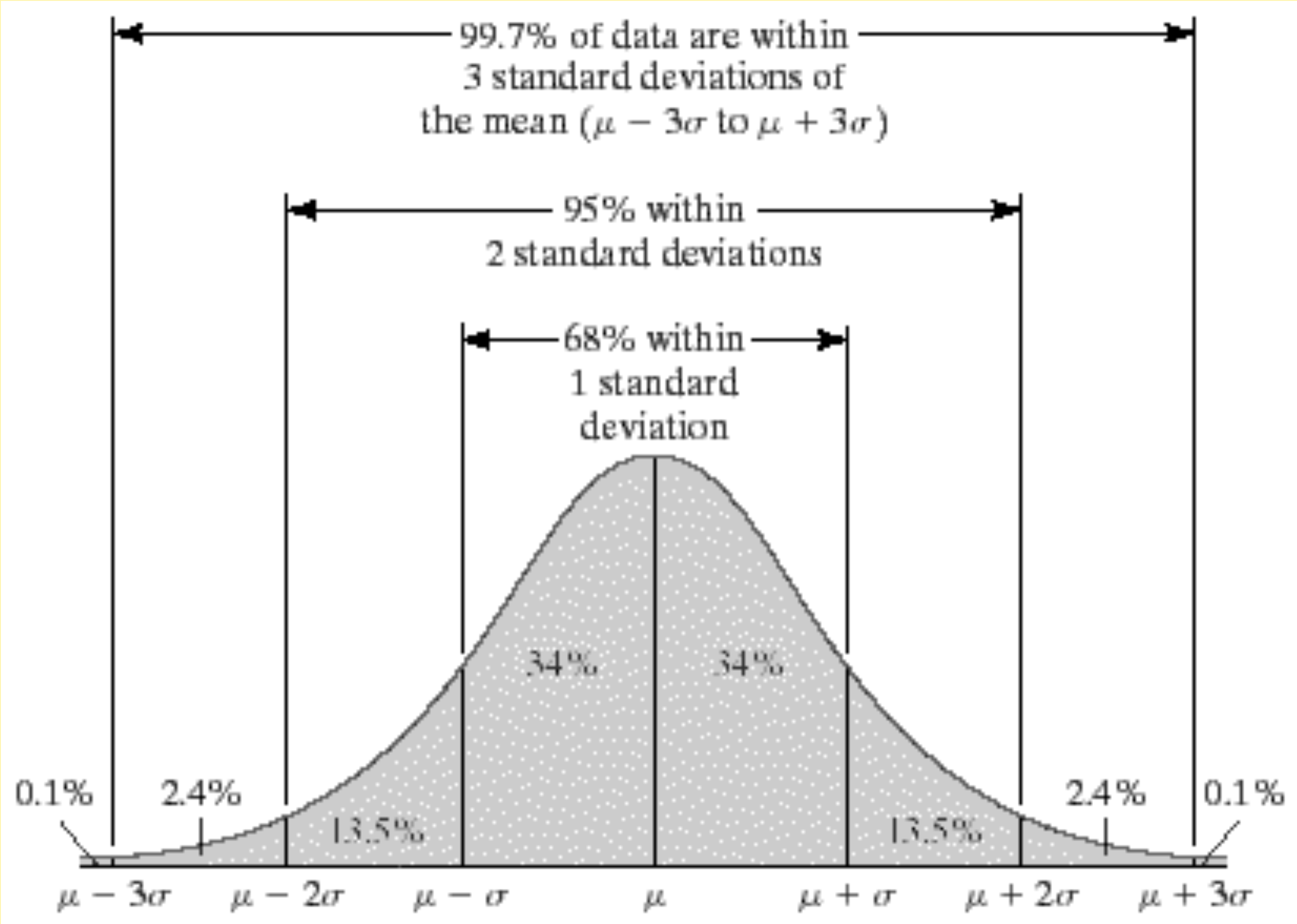


- Carl Friedrich Gauß (1777-1855), ***Gaussian Distribution***
- Normal distribution is perfectly ***symmetric and bell-shaped***
- Characterized by two parameters: ***mean μ*** and ***standard deviation σ***
- The ***68%-95%-99.7% rule*** applies to the normal distribution; that is, the probability concentrated within 1 standard deviation of the mean is always 0.68; within 2, 0.95; within 3, 0.997.
- The ***IQR ≈ 4/3σ rule*** also applies

Standard Deviation Interpretation: Empirical Rule

- ***If*** the histogram of the data is **approximately symmetric and bell-shaped**, then
  - About **68% of the data are within one** standard deviation from the mean
  - About **95% of the data are within two** standard deviations from the mean
  - About **99.7% of the data are within three** standard deviations from the mean

Standard Deviation Interpretation: Empirical Rule



Normal Distribution Example

- Female Heights: women between the ages of 18 and 24 average 65 inches in height, with a standard deviation of 2.5 inches, and the distribution is approximately normal.
- Choose a woman of this age at random: the probability that her height is between μ-σ=62.5 and μ+σ=67.5 inches is ______%?
- Choose a woman of this age at random: the probability that her height is between μ-2σ=60 and μ+2σ=70 inches is ______%?
- Choose a woman of this age at random: the probability that her height is greater than μ+2σ=70 inches is ______%?

Normal Distributions

- So far, we have looked at the probabilities within one, two, or three standard deviations from the mean

$$(\mu \pm \sigma, \mu \pm 2\sigma, \mu \pm 3\sigma)$$

- How much probability is concentrated within 1.43 standard deviations of the mean?
- More generally, how much probability is concentrated within *z* standard deviations of the mean?

Calculation of Normal Probabilities

Applet or Z table:

Gives amount of probability between 0 and *z*, the *standard normal* random variable.

So what about the "*z* standard deviations of the mean" stuff from last slide?

Normal Distribution Table

- Z Table or applet shows, for different values of z, the probability to the left of $\mu + z\sigma$ (the cumulative probability)

- Probability that a normal random variable takes any value up to z standard deviations above the mean

- For z =1.43, the tabulated value is .9236

- That is, the probability **less than or equal to $\mu + 1.43\sigma$** for a normal distribution equals .9236

Why the table with Standard Normal Probabilities is all we Need

- When values from an arbitrary normal distribution are converted to *z*-scores, then they have a standard normal distribution
- The conversion is done by subtracting the mean μ, and then dividing by the standard deviation σ:

$$z = \frac{x - \mu}{\sigma}$$

z-scores: properties and uses

- The z-score for a value x of a random variable is the number of standard deviations that x is above μ

- If x is below μ, then the z-score is negative

- The z-score is used to compare values from different (normal) distributions

z-scores: properties and uses

- The z-score is used to compare values from different normal distributions
- SAT: $\mu = 500$, $\sigma = 100$
- ACT: $\mu = 18$, $\sigma = 6$
- Which is better, 650 in the SAT or 25 in the ACT?

$$z_{\text{SAT}} = \frac{650 - 500}{100} = 1.5 \qquad z_{\text{ACT}} = \frac{25 - 18}{6} = 1.17$$

Backwards *z* Calculations

- We can also use the table to find *z-values* for given probabilities
- Find the *z-value* corresponding to a right-hand tail probability of 0.025
- This corresponds to a probability of 0.975 to the left of z standard deviations above the mean
- Table: $z = 1.96$

Going in Reverse, S'More

- Find the *z-value* for a right-hand tail probability

  – of 0.1 is z = __________.

  – of 0.01 is z = __________.

  – of 0.05 is z = __________.

Example

- The weekly amount spent for maintenance and repair in a certain company has an approximately normal distribution with a mean of $400 and a standard deviation of $20. If $450 is budget to cover repairs for next week, what is the probability that the actual costs will exceed the budget amount?