STA 321 Spring 2014

Lecture 16 *Thursday, March 27*

- **Hypothesis Tests**

Significance Test

- A ***significance test*** is a way of statistically testing a hypothesis by comparing the data to values predicted by the hypothesis
- Data that fall far from the predicted values provide ***evidence against the hypothesis***

Elements of a Significance Test

- Assumptions
- Hypotheses
- Test Statistic
- P-value
- Conclusion

Assumptions

- What type of data do we have?
  - Qualitative or quantitative?
  - Different types of data require different test procedures
- What is the population distribution?
  - Is it normal? Symmetric?
  - Some tests require normal population distributions
- Which sampling method has been used?
  - We always assume simple random sampling
  - Other sampling methods are discussed in STA 675
- What is the sample size?
  - Some methods require a minimum sample size (like $n=30$)

Hypotheses

- The ***null hypothesis*** (H_0) is the hypothesis that we test (and try to find evidence against)
- The name null hypothesis refers to the fact that it often (not always) is a hypothesis of "no effect" (no effect of a medical treatment, no difference in characteristics of countries, etc.)
- The ***alternative hypothesis*** (H_a) is a hypothesis that contradicts the null hypothesis
- When we reject the null hypothesis, the alternative hypothesis is judged acceptable
- Often, the alternative hypothesis is the actual research hypothesis that we would like to "prove" by finding evidence against the null hypothesis (proof by contradiction)

Hypotheses

The hypothesis is always a statement about one or more population parameters.

Test Statistic

- The ***test statistic*** is a statistic that is calculated from the sample data
- Often, the test statistic involves a point estimator of the parameter about which the hypothesis is stated
- For example, the test statistic may involve the sample mean or sample proportion if the hypothesis is about the population mean or population proportion

P-Value

- How unusual is the observed test statistic when the null hypothesis is assumed true?
- The ***P-value*** is the probability, assuming that H_0 is true, that the test statistic takes values at least as contradictory to H_0 as the value actually observed
- The smaller the P-value, the more strongly the data contradict H_0

Conclusion

- In addition to reporting the P-value, a formal decision is made about rejecting or not rejecting the null hypothesis
- Most studies choose a cutoff of 5%.
- This corresponds to rejecting the null hypothesis for P-values smaller than 0.05.
- Smaller P-values provide more significant evidence against the null hypothesis
- “The results are significant at the 5% level”

Elements of a Significance Test

- Assumptions
  - Type of data, population distribution, sample size
- Hypotheses
  - Null and alternative hypothesis
- Alpha-level (Type I error probability)
  - Specify alpha-level before looking at data
  - Alpha-level determines rejection region
- Test Statistic
  - Compares point estimate to parameter value under the null hypothesis
- P-value
  - Uses sampling distribution to quantify evidence against null hypothesis
  - Small P is more contradictory
- Conclusion
  - Report P-value
  - Rejection if test statistic in rejection region or P-value < alpha-level

P-Value

- How unusual is the observed test statistic when the null hypothesis is assumed true?
- The ***p-value*** is the probability, assuming that H_0 is true, that the test statistic takes values at least as contradictory to H_0 as the value actually observed
- *The **p-value** is **not** the probability that the hypothesis is true*
- The smaller the *p*-value, the more strongly the data contradict H_0

Alpha-Level

- Alpha-level (significance level) is a number such that one rejects the null hypothesis if the *p*-value is less than or equal to it.
- Often, alpha=0.05
- Choice of the alpha-level reflects how cautious the researcher wants to be
- Significance level alpha needs to be chosen ***before*** analyzing the data

Rejection Region

- The rejection region is a range of values such that if the test statistic falls into that range, we decide to reject the null hypothesis in favor of the alternative hypothesis

Type I and Type II Errors

- Type I Error: The null hypothesis is rejected, even though it is true.
- Type II Error: The null hypothesis is not rejected, even though it is false.

Type I and Type II Errors

- Terminology:
  - ***Alpha*** = Probability of a Type I error
  - ***Beta*** = Probability of a Type II error
  - ***Power*** = 1 – Probability of a Type II error
- The smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- If you ask for very strong evidence to reject the null hypothesis, it is more likely that you fail to detect a real difference

Type I and Type II Errors

- In practice, alpha is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult
- **How to choose alpha?**
- If the consequences of a Type I error are very serious, then alpha should be small.
- For example, you want to find evidence that someone is guilty of a crime
- In exploratory research, often a larger probability of Type I error is acceptable
- If the sample size increases, both error probabilities decrease

Power Calculations

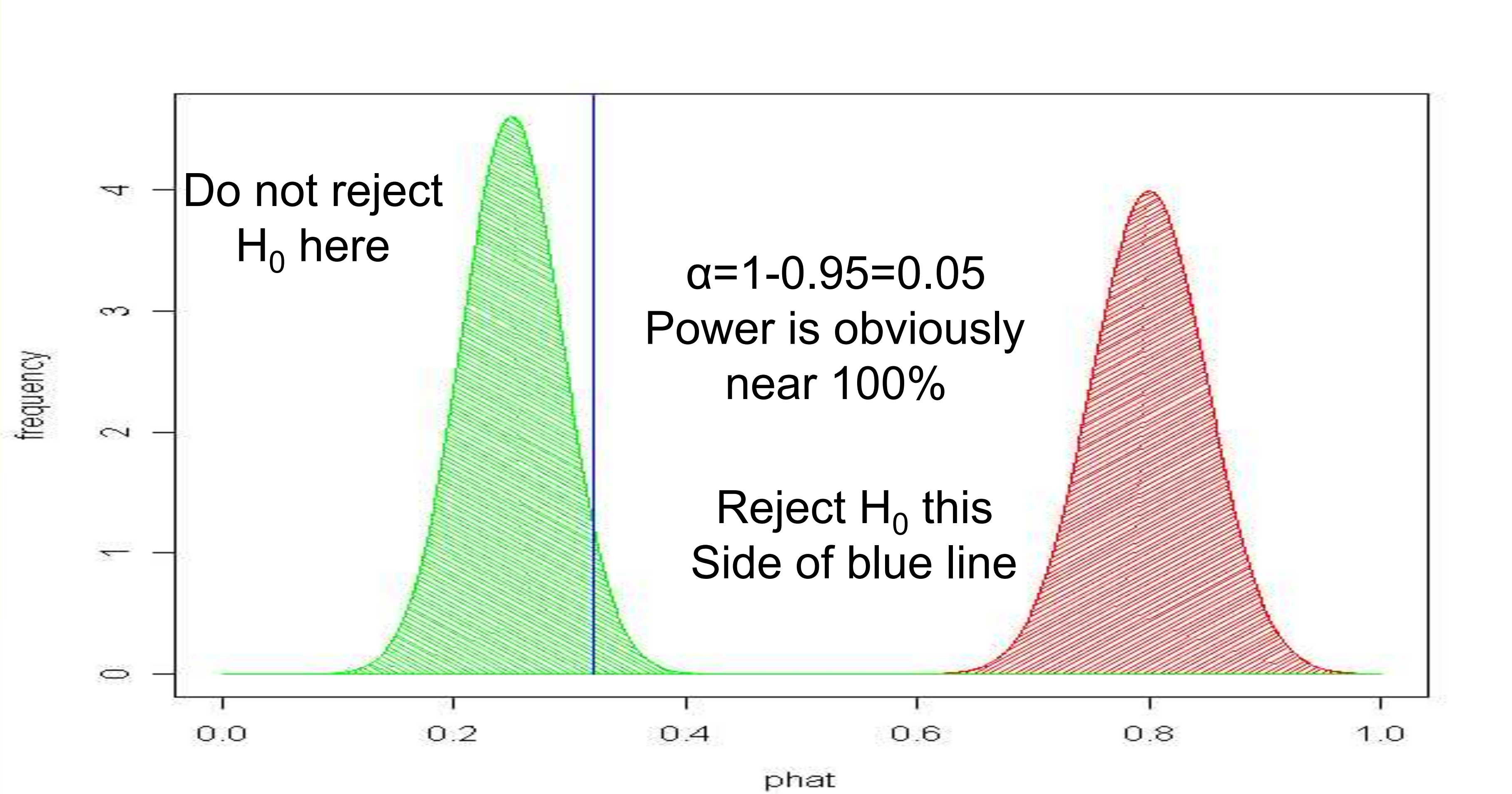
Recall our ESP Example

- Hypothesis H_0 : p=0.25 against
  H_1 : p>0.25.
- The null distribution is normal with mean 0.25 and standard deviation sqrt (0.25*0.75/n)
- The cutoff is the 1-α percentile of this null distribution. We reject H_0 if p-hat is above this cutoff.

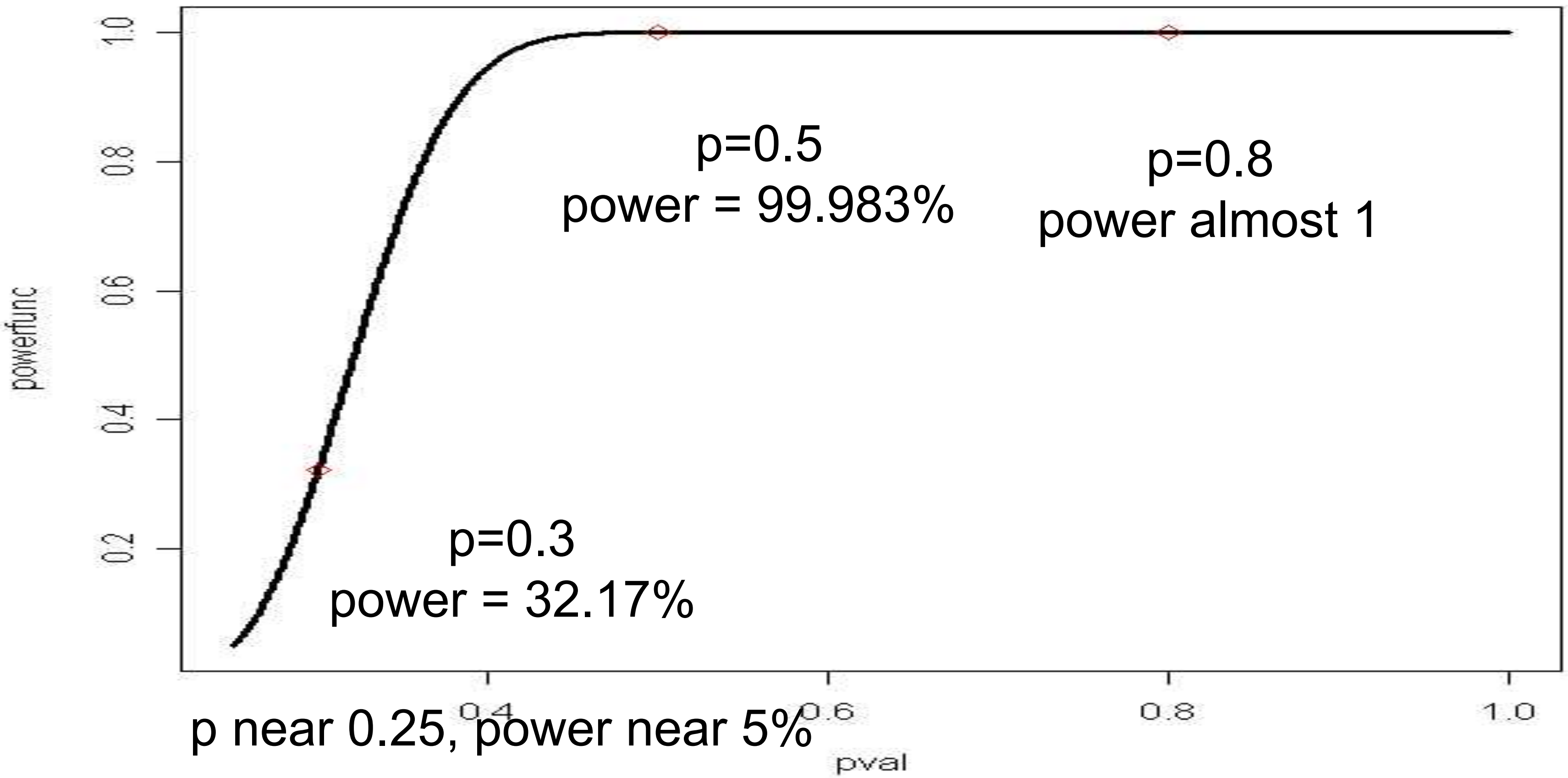
Power Function

- What about the power of the test?
- Unfortunately under H_1 we know nothing more about p than $p>0.25$
- BUT we can compute the power for ***each*** $p>0.25$

What if the Alternative Is H_1:p=0.8?



Power Function



Questions…

- What happens to the power function when we use $\alpha=0.001$ instead of $\alpha=0.05$?
- What happens to the power function when the sample size n is increased?
- How can we be sure we get a particular amount of power in our experiment?

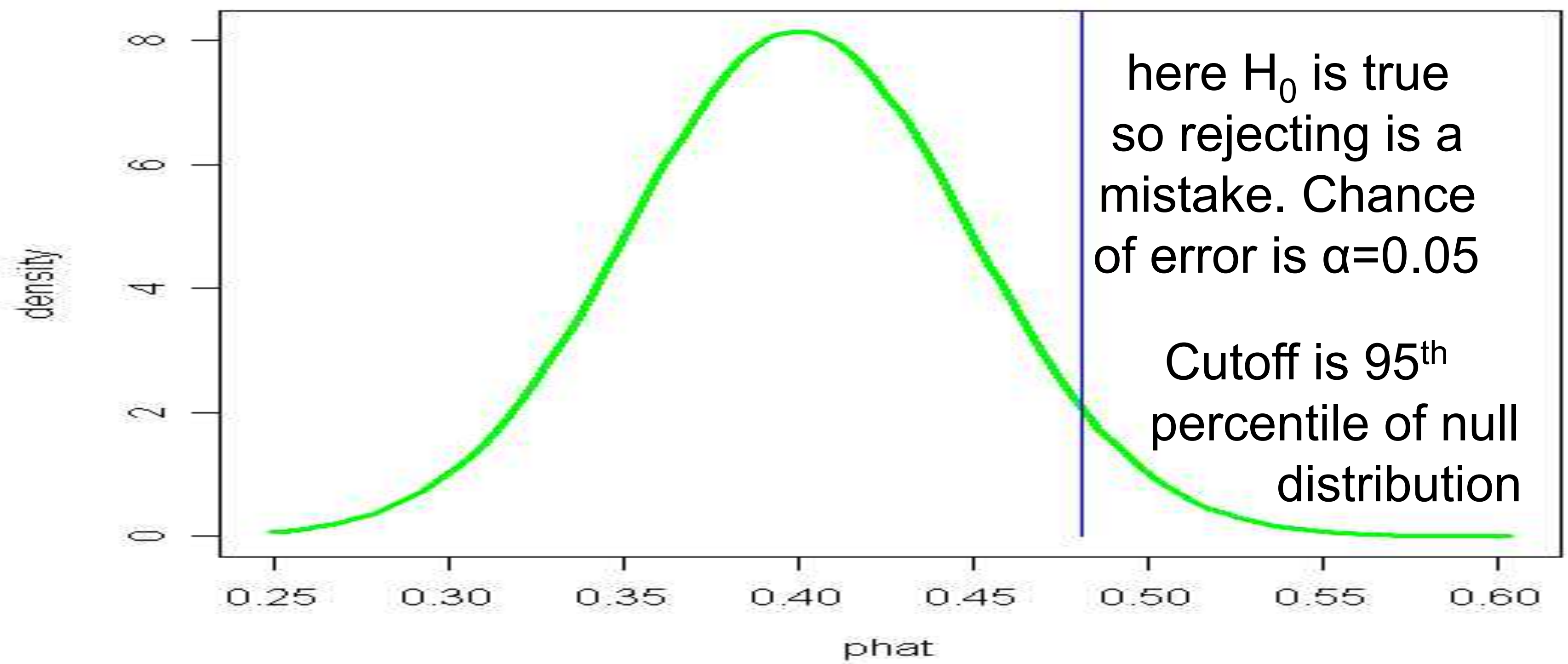
Power – Why Important?

- Example
  - A pharmaceutical company knows that the old treatment for a disease cures 40% of the people.
  - They hope their new treatment is better.
  - They hope they can get the cure rate to 45%.

Power Example Continued

- Our hypothesis test will test the null hypothesis H_0 : p=0.4 (the old treatment proportion) against H_1 : p>0.4 (we want our treatment to do better, hence this alternative).
- We intend to give the treatment to 100 people, and using α=0.05
- Cutoff?
- 95^{th} percentile of the null distribution.
- Z=1.64, thus Y = (0.049)(1.64) + 0.4 = 0.4804

Null Distribution Centered at p_0=0.4



here H_0 is true so rejecting is a mistake. Chance of error is α=0.05

Cutoff is 95th percentile of null distribution

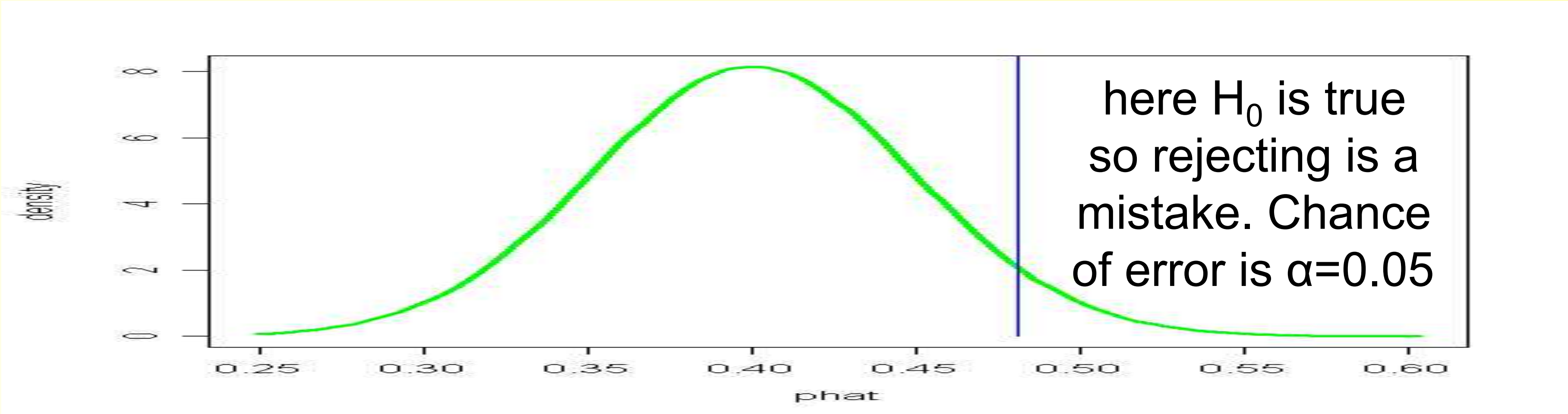
do not reject this side of blue line

reject this side of blue line

Can this Experiment Find Anything?

- We only are ***guessing*** our treatment can get the cure rate to 45%.
- What is the power for 45%?
- Remember, the power is the chance that, ***when p=0.45***, we reject H_0 (the right decision in that case).
- We reject when p-hat>0.4804.
- Thus, we need the probability that p-hat is greater than 0.4804, ***given*** p=0.45.

Green curve is distribution for p=0.4
Red curve is distribution for p=0.45



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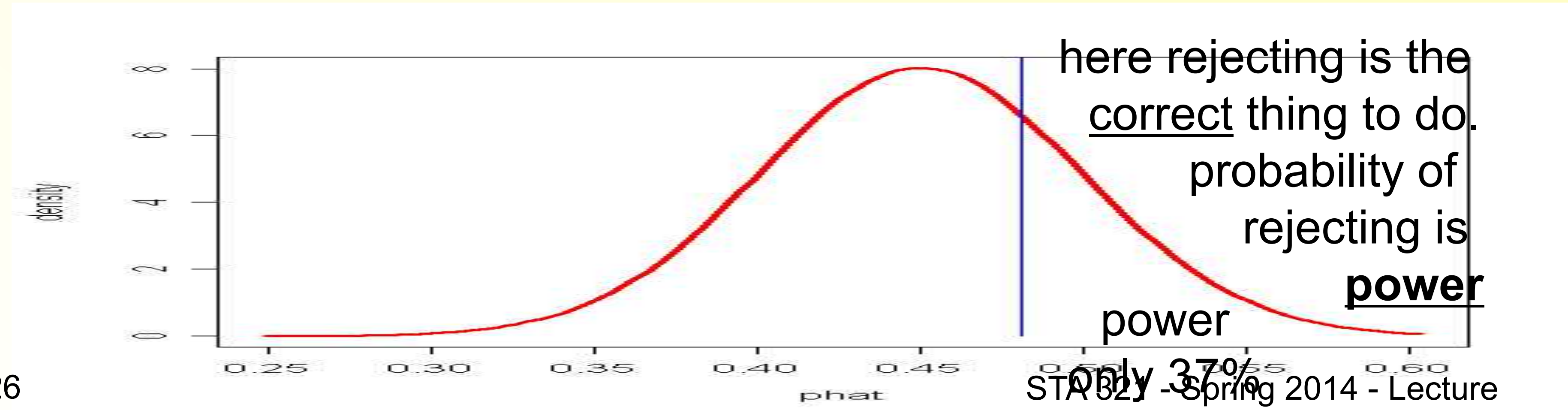

here rejecting is the <u>correct</u> thing to do. probability of rejecting is **<u>power</u>**

power only 37%

What This Means…

- Suppose our treatment works (this is the assumption under which the power is calculated).
- Then we only have a 37.09% chance of getting a “reject H_0” conclusion.
- That is not great. We could have a beneficial treatment and miss it.
- Solution – choose a higher sample size.

Power Equation

- We need to solve for the n that satisfies

$$\frac{0.4899(1.645)}{\sqrt{n}} + 0.4 = \frac{0.4975(-1.28)}{\sqrt{n}} + 0.45$$

$$\frac{1.4427}{\sqrt{n}} = 0.05$$

- n=832.5, so n must be at least 833