

STA 321

Spring 2014

Lecture 17

Tuesday, April 1

➤ **Hypothesis Tests**

Another Example

- A person claims to have ESP.
- You test him/her by having one of four images displayed on a computer screen to a second individual.
- The individual claiming to have ESP has to guess what is on the computer screen.
- Just by random guessing, you would expect the “ESP person” to get 25% of the images correct.
- Thus, a claim of ESP would be strengthened if the person guessed more than 25% of the images correctly.

ESP Example in Statistical Terms

- Suppose you show the person $n=100$ images. Each guess is correct or incorrect (binary, dichotomous).
- The proportion of correct guesses is \hat{p} .
- The true proportion of correct guesses (unknown to us) is p .
- Under random guessing, $p=0.25$
- If the person has ESP, $p>0.25$

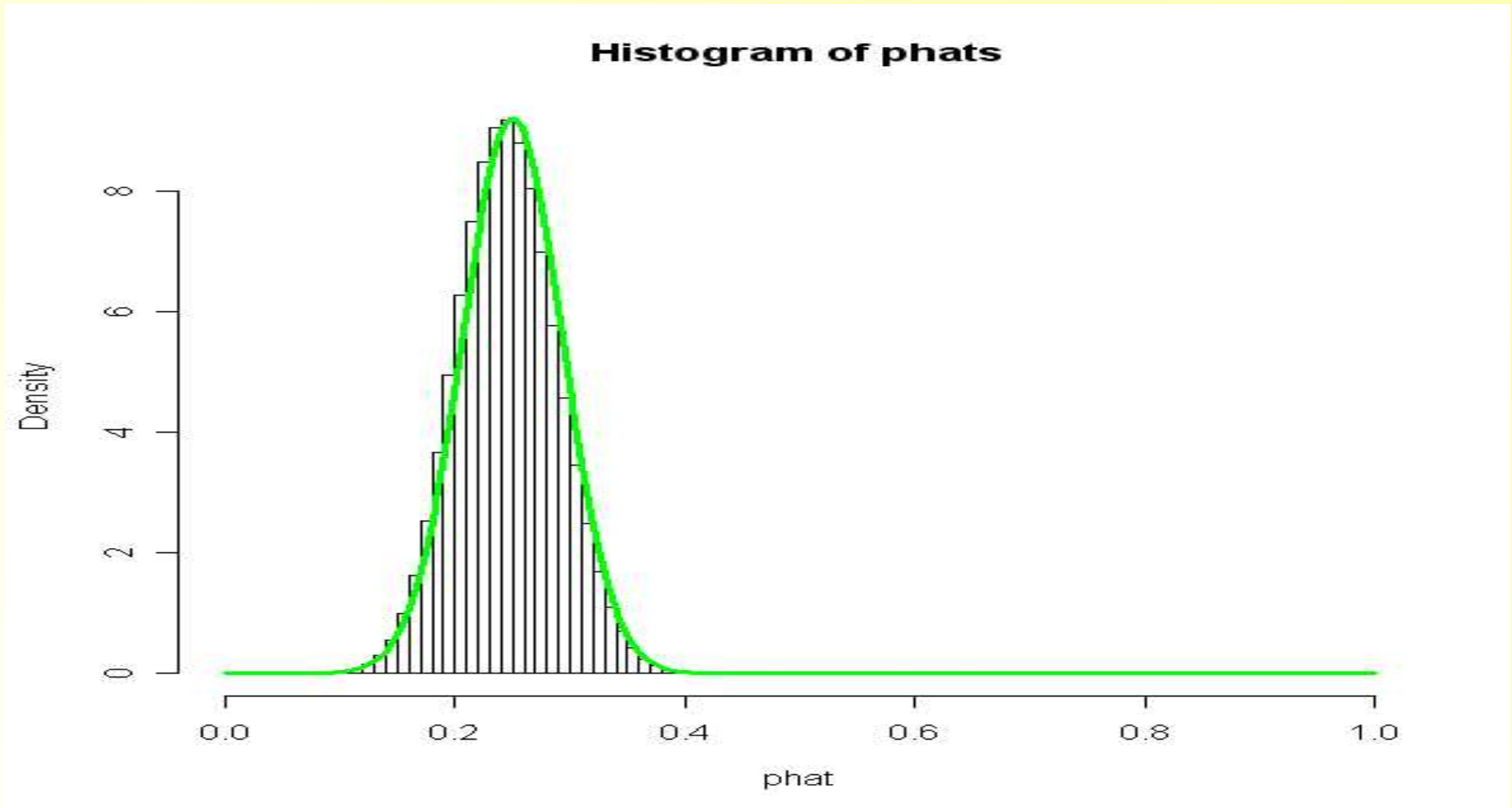
Null and Alternative Hypotheses

- The cutoff is based on the null hypothesis.
- $H_0 : p=0.25$ allows us to find a sampling distribution.
- The hypothesis “ $p>0.25$ ” would not determine a “ p ” and a sampling distribution.
- It is a ***composite, one-sided alternative***.
- The alternative hypothesis is the “interesting conclusion” which would get the paper published.

Null Distribution

- Under the null hypothesis, $p=0.25$.
- $n=100$.
- Thus, the null distribution is normal with mean 0.25 and standard deviation $\sqrt{(0.25*0.75/100)}=0.0433$.
- The null distribution here is the distribution of correct guesses under random guessing.

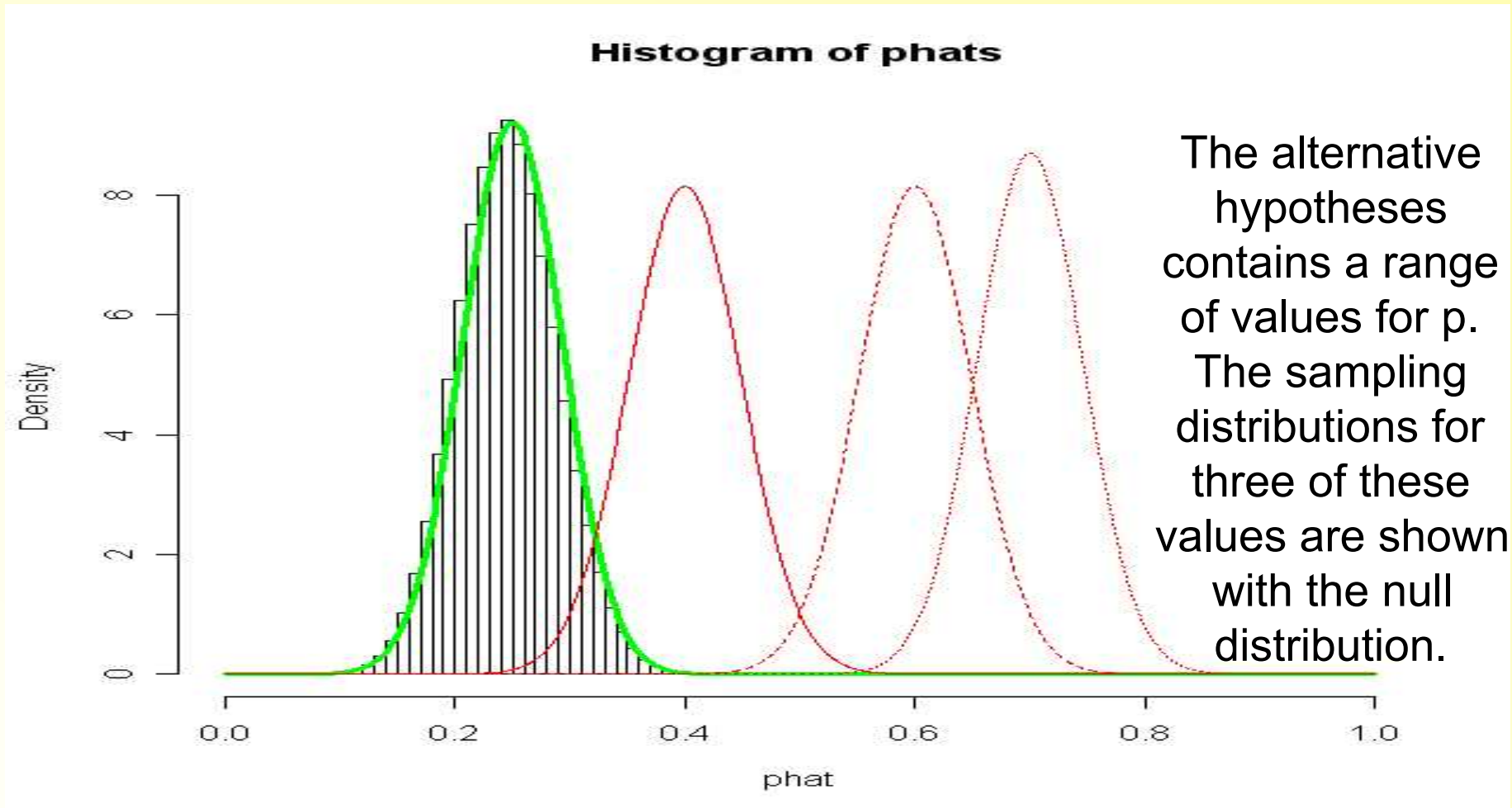
Sampling Distribution Under $H_0 : p=0.25$



Difficulty of Composite Alternative Hypotheses

- Finding the sampling distribution under H_1 is impossible.
- $H_1 : p > 0.25$ only specifies a range for p .
- The true proportion p might be 0.8, it might be 0.3, it might be 0.99, we don't know.
- This is different from the “Bill vs. Maria” where the two possible models were exactly specified.

Any of the Red Curves Could Be the Alternative Distribution



Where to Place the Cutoff?

- The alternative hypothesis is designed to be the “interesting conclusion”, thus the statistical test is designed to avoid mistakenly choosing H_1 .
- Thus, we give H_0 the “benefit of the doubt” by choosing a small probability of type I error.

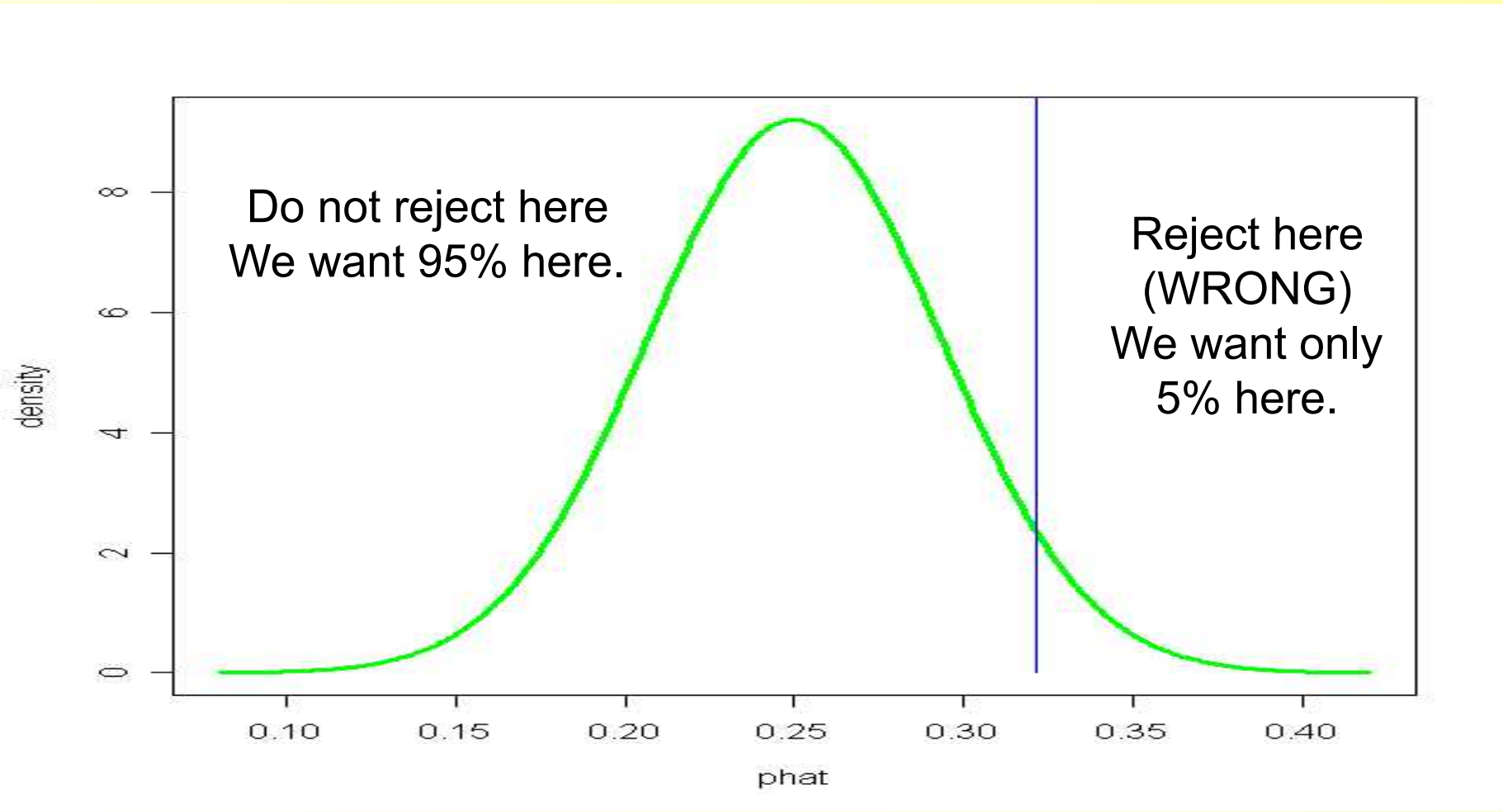
Why Give H_0 the Benefit of the Doubt?

- In the ESP example, we probably would not be convinced if someone did just a little better than 25%.
- While 25% is the expected number of correct guesses, you could do a little better based on chance alone.
- So how much better does the person have to do before you'd start to believe them?

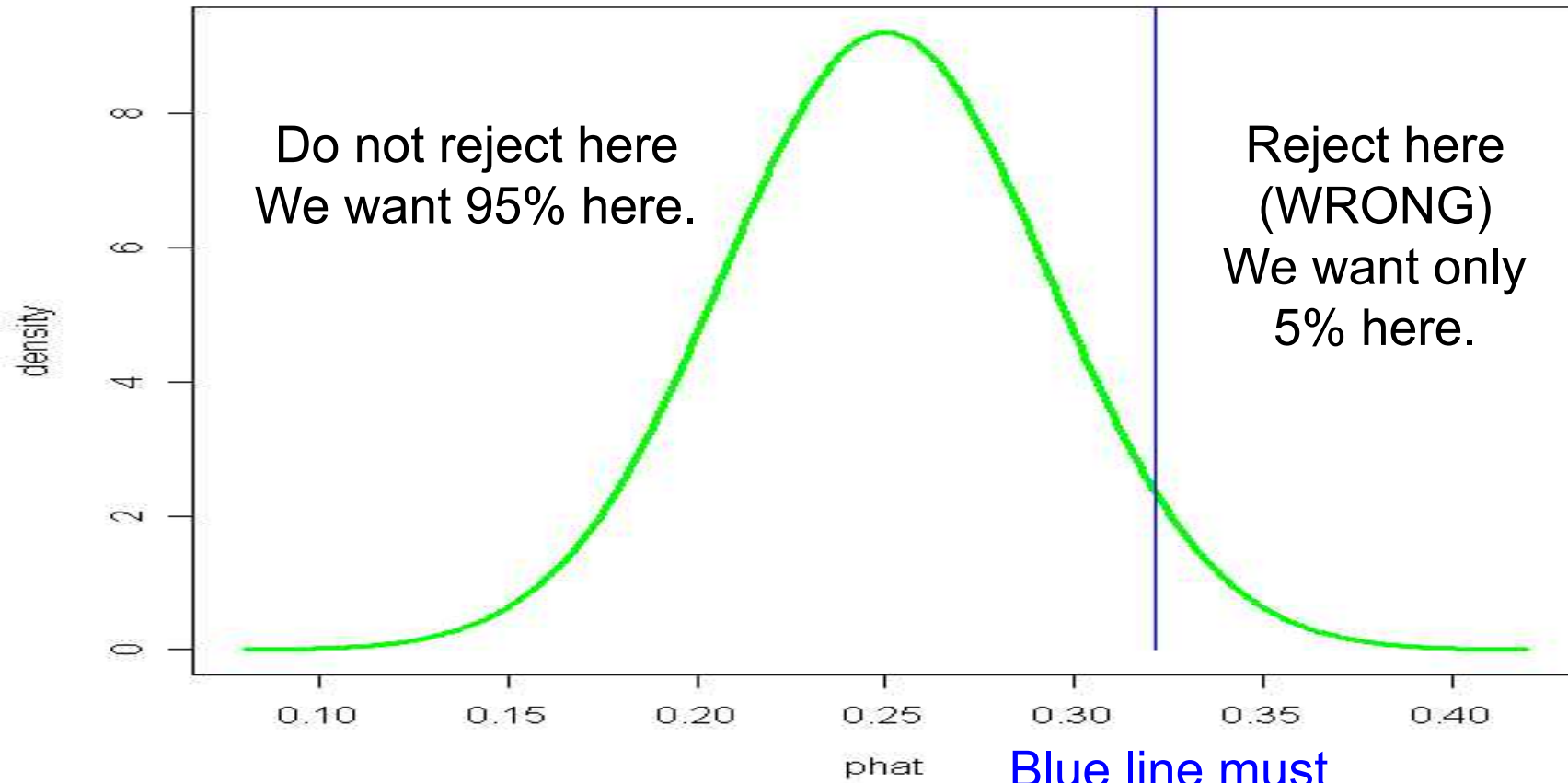
Finding the Cutoff

- Again let's choose $\alpha=0.05$
- The null hypothesis contains $p=0.25$ while the alternative hypothesis contains $p>0.25$. Thus, we reject H_0 for large values of \hat{p} .
- We need 95% of the null distribution below the cutoff.

Null Distribution



Null Distribution

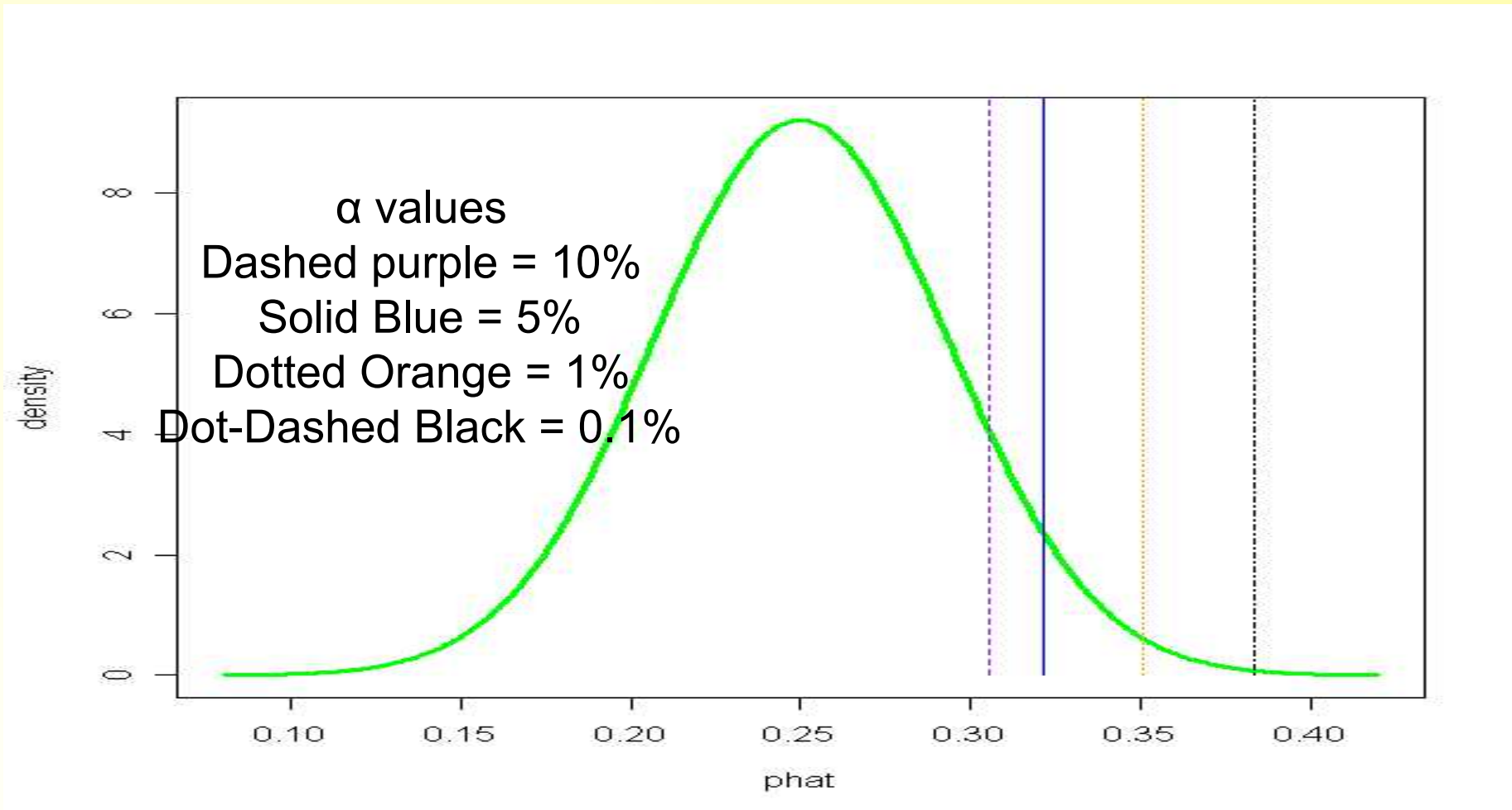


Blue line must
be the 95th
percentile

Finding the 95th Percentile

- The null distribution is normal with mean 0.25 and standard deviation 0.0433
- The corresponding Z-score for the 95th percentile is 1.64 and thus the cutoff is $(1.64)(0.0433)+0.25=0.3210$.
- This cutoff says:
 - Getting a little more than 25% correct out of 100 isn't enough to convince us.
 - Our benefit of the doubt says we don't believe H_1 until you get at least 32.10% correct.

Different α Values Result in Different Cutoffs



Type I and Type II Errors

	Choose H_0 (“ <u><i>do not reject H_0</i></u> ”)	Choose H_1 (“ <u><i>reject H_0</i></u> ”)
H_0 is true	Correct answer	WRONG! (<u>Type I error</u>)
H_1 is true	WRONG! (<u>Type II Error</u>)	Correct answer

Type I and Type II Errors

- We reject $p=0.25$ if \hat{p} is greater than 0.3210.
- The chance of making a type I error (rejecting H_0 when H_0 is in fact true) is 0.05.
- Calculating the type II error probability is more complicated.
- For each different value in the alternative, the type II error probability is different.
- For values close to 0.25 (e.g., 0.26), the type II error probability is high because the sampling distribution overlaps much with the null distribution
- For values far from 0.25 (e.g., 0.9), the probability of type II error is low.

More on Type II Error

- One issue that we will study is how many observations should we collect to achieve particular error probabilities.
- *Power and sample size calculations.*

Review

- We have n observations. The true proportion, p , is unknown.
- We have a null value of $H_0 : p=p_0$, and depending on the setting may want to test $H_1 : p>p_0$ or $H_1 : p<p_0$, or $H_1 : p\neq p_0$.
- The alternative depends on the “scientifically interesting” conclusion.
- It may only matter if p is above p_0 , or p below p_0 , or perhaps simply being different.

Review

- We use the data (specifically \hat{p} , the sample proportion) to make one of two conclusions.
- One conclusion is to reject H_0 , indicating that the data are not consistent with H_0 .
- The other possible conclusion is to “not reject H_0 ”, which indicates the data is consistent with H_0 .
- *This is not the same as saying H_0 is true, since it is impossible to distinguish, with finite data, “ $p=p_0$ ” from “ p very close to p_0 .”*

Significance Tests: Summary

- A significance test checks whether data agrees with a hypothesis
- A hypothesis is a statement about a characteristic of a variable or a collection of variables
- If the data is very unreasonable under the hypothesis, then we will reject the hypothesis
- Usually, we try to find evidence ***against*** the hypothesis

Logical Procedure

1. State a hypothesis that you would like to find evidence against
2. Get data and calculate a statistic (for example: sample mean)
3. The hypothesis (for example: population mean equals 5) determines the sampling distribution of our statistic
4. If the calculated value in 2. is very unreasonable given 3., then we conclude that the hypothesis was wrong

Significance Test

- A ***significance test*** is a way of statistically testing a hypothesis by comparing the data to values predicted by the hypothesis
- Data that fall far from the predicted values provide ***evidence against the hypothesis***

Elements of a Significance Test

- Assumptions
- Hypotheses
- Test Statistic
- P-value
- Conclusion

Assumptions

- What type of data do we have?
 - Qualitative or quantitative?
 - Different types of data require different test procedures
- What is the population distribution?
 - Is it normal? Symmetric?
 - Some tests require normal population distributions
- Which sampling method has been used?
 - We always assume simple random sampling
 - Other sampling methods are discussed in STA 675
- What is the sample size?
 - Some methods require a minimum sample size (like $n=30$)

Hypotheses

- The ***null hypothesis*** (H_0) is the hypothesis that we test (and try to find evidence against)
- The name null hypothesis refers to the fact that it often (not always) is a hypothesis of “no effect” (no effect of a medical treatment, no difference in characteristics of countries, etc.)
- The ***alternative hypothesis*** (H_a) is a hypothesis that contradicts the null hypothesis
- When we reject the null hypothesis, the alternative hypothesis is judged acceptable
- Often, the alternative hypothesis is the actual research hypothesis that we would like to “prove” by finding evidence against the null hypothesis (proof by contradiction)

Hypotheses

The hypothesis is always a statement about one or more population parameters.

Test Statistic

- The ***test statistic*** is a statistic that is calculated from the sample data
- Often, the test statistic involves a point estimator of the parameter about which the hypothesis is stated
- For example, the test statistic may involve the sample mean or sample proportion if the hypothesis is about the population mean or population proportion

P-Value

- How unusual is the observed test statistic when the null hypothesis is assumed true?
- The ***P-value*** is the probability, assuming that H_0 is true, that the test statistic takes values at least as contradictory to H_0 as the value actually observed
- The smaller the P-value, the more strongly the data contradict H_0

Conclusion

- In addition to reporting the P-value, a formal decision is made about rejecting or not rejecting the null hypothesis
- Most studies choose a cutoff of 5%.
- This corresponds to rejecting the null hypothesis for P-values smaller than 0.05.
- Smaller P-values provide more significant evidence against the null hypothesis
- “The results are significant at the 5% level”

Multiple choice

If a test of hypothesis has a Type I error probability of .05, this means that:

- a. if the null hypothesis is true, we don't reject if 5% of the time.
- b. if the null hypothesis is true, we reject it 5% of the time.
- c. if the null hypothesis is false, we don't reject it 5% of the time.
- d. if the null hypothesis is false, we reject it 5% of the time.