Name_____

STA 321 Midterm 1 Practice Exam

Basic Statistical Theory I

March 8th, 2016

There are five questions on this test. DO use calculators if you need them. "And then a miracle occurs" is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 75 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: Central Limit Theorem The weights of adults males are normally distributed with a mean of 172 pounds and a standard deviation of 29 pounds (based on data from the National Health Survey). Use this information to answer parts (a) through (c).

(a) What is the probability that one randomly selected adult male will weight more than 190 pounds?

(b) What is the probability that 25 randomly selected adult males will have a mean weight of more than 190 pounds?

(c) An elevator at a men's fitness center has a sign that says the maximum allowable weight is 4750 pounds. If 25 randomly selected men cram into the elevator, what is the probability it will be over the maximum allowable weight?

Question 2: Bayes Estimators

The number of defects in a magnetic tape has a Poisson distribution with unknown mean θ. The prior distribution of θ is a gamma distribution with $\alpha = 3$, $\beta = 1$. Five rolls of magnetic tape are tested for defects and it is found that the number of defects is 2, 2, 6, 0, 3. If we use the squared error loss function, what is the Bayes estimate of θ?

Question 3: MLE

Suppose that $X_1, \ldots, X_n$ form a random sample from the geometric distribution, which is a discrete distribution with probability function

$$f(x|\theta) = \theta(1-\theta)^x,$$

for $x = 0, 1, 2, \ldots$. where $0 < \theta < 1$ is the (unknown) parameter of the distribution. Obtain the M.L.E. of θ. Describe the scenario for the observed data values, $x_1, \ldots, x_n$, under which the M.L.E. of θ does not exist.

Questions 4: Confidence Intervals The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes.

(a) After observing 120 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a 92% confidence interval for the mean assembly time.

(b) How many workers should be involved in this study in order to have the mean assembly time estimated up to ± 15 seconds with 92% confidence?