Lecture notes for STA 321 for day 2 by R. Yoshida

1 Sec 6.2: The law of large numbers (part II)

As we saw from the last lecture as we increase the sample size n, the sample mean $\bar{X}_n$ is getting closer to μ. The law os large numbers states this phenomena mathematically.

Definition 1 (Convergence in probability)**.** *A sequence of random variables $Z_1, Z_2, \ldots$ is said to convergence in probability to $b \in \mathbb{R}$ is for any $\epsilon > 0$*

$$\lim_{n \to \infty} P(|Z_n - b| < \epsilon) = 1.$$

We notate this as

$$Z_n \xrightarrow{p} b.$$

Theorem 2 (Law of large numbers)**.** *Suppose $X_1, \ldots, X_n$ form a random sample from a distribution with its mean μ and variance $\sigma^2 < \infty$, then*

$$\bar{X}_n \xrightarrow{p} \mu.$$

Proof. We have $\sigma^2 < \infty$.

□

Theorem 3. *If $Z_n \xrightarrow{p} b$ and g is a continous function, then*

$$g(Z_n) \xrightarrow{p} g(b).$$

Proof. Its proof is a HW. □

Theorem 4. *Let $X_1, \dots X_n$ form a random sample and let $c_1 < c_2$, where $c_1, c_2 \in \mathbb{R}$. Define*

$$Y_i = \begin{cases} 1 & \text{if } c_1 \leq X_i < c_1, \\ 0 & \text{otherwise.} \end{cases}$$

Then,

$$\bar{Y}_n \xrightarrow{p} P(c_i \leq X_i < c_2).$$

Proof. Note that

□

Remark. We can use this to the histogram.

Example 5. *Tossing a coin n timees with $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p$, where $0.1 < p < 0.9$ and*

$$X_i = \begin{cases} 1 & \text{if we observe a head,} \\ 0 & \text{otherwise.} \end{cases}$$

Let $p = 1/2$.

Now we will show some simulations for the histogram.