Lecture notes for STA 321 for day 8 by R. Yoshida

1 Sec 7.4: Bayes Estimator

Now we difine the *estimate* of θ.

Definition 1. *Let $X_1, \ldots, X_n$ be the observable data whose joint distribution is indexed by a parametor $\theta \in \Omega \subset \mathbb{R}$. An* estimator *of the parameter θ is a real valued function $\delta(X_1, \ldots, X_n)$, If $X_1 = x_1, \ldots, X_n = x_n$ are observed data then $\delta(x_1, \ldots, x_n)$ is called the* estimate *of θ.*

Remark 2. *Since the estimator of θ is a function of the data it is a statistics.*

Example 3. *A store owner models the number of customers arriving at the store by Poisson distribution with unknown rate θ. The owner assigns the distribution of θ a gamma prior distribution with parameter $\alpha = 3$ and $\beta = 2$. Let X be the number of customers during one hour. If the data $X_1 = 3, X_2 = 4, X_3 = 2, X_4 = 2, X_5 = 3, X_6 = 5, X_7 = 6, X_8 = 1$ is observed, what is the distribution of θ given the data?*

Definition 4. *A* loss function $L(\theta, a)$ *is a real valued function of two variables* $\theta \in \Omega$ *and* $a \in \mathbb{R}$. *The intuitive meaning of the loss function is*

Definition 5. *Let* $L(\theta, a)$ *is a loss function. For any* $X = x$, *let* $\delta^*(x)$ *be the value such that* $\mathbb{E}[l(\theta, a)|x]$ *is minimized. Then* $\delta^*(x)$ *is called the* Bayes estimator *of* $\theta \in \Omega$, *namely:*

$$\delta^*(x) = \arg\min_a \int_{\theta \in \Omega} L(\theta, a)\xi(\theta|x)d\theta$$

where $\xi(\theta|x)$ *is the pdf for the posterior distribution.*

Definition 6. *If* $L(\theta, a) = (\theta - a)^2$, *then it is called the* squared error loss.

Theorem 7. *If* $L(\theta, a) = (\theta - a)^2$, *then the Bayes estimator is*

$$\delta^*(X) = \mathbb{E}[\theta|X].$$

Proof. We are finding the optimal solution by...

□

Example 8. *Go back to the example...*