Lecture notes for STA 321 for day 9 by R. Yoshida

1 Sec 7.5: Maximum likelihood estimators

Suppose we have a random sample $\{X_1, \ldots, X_n\}$ with a conditional ppf or pdf $f(x|\theta)$. The goal of the maximum likelihood estimator (MLE) is to estimate θ by optimizing the joint conditional pdf or pf $f_n(x|\theta)$.

Definition 1. *The conditional joint pf or pdf $f_n(x|\theta)$ of the observations in a random sample is called a* likelihood function *of θ if we consider $L = (\theta) = f_n(x|\theta)$ as a function of θ.*

Remark 2. *$L(\theta)$ is not a pdf or pf in terms of θ.*

Definition 3. *For each observation $X \in \mathbb{R}^n$ let $\delta(X) \in \Omega$ be the optimal value $\hat{\theta}$ such that $L(\hat{\theta})$ takes the maximum. Then the estimator $\hat{\theta}$ is called a* maximum likelihood estimator (MLE) *of θ. For each observation $X = x$, $\delta(x)$ is called a* maximum likelihood estimate *of θ.*

Example 4. *We model the like time of a light bulb by the exponential distribution with the unknown rate $\theta > 0$. Suppose X_i is the time in terms of a year that the ith light bulb is out. SUppose we have the following observation:*

$$X_1 = 3,\ X_2 = 1.5,\ X_3 = 2.1,\ X_4 = 1.9,\ X_5 = 1.6,\ X_6 = 1.1,\ X_7 = 2.5.$$

Compute the likelihood function of θ from this observation.

Definition 5. *A function $\ell(\theta)$ is called a* log likelihood function *of θ if*

$$\ell(\theta) = L(\theta).$$

Example 6. *Go back to the previous example....*

Example 7. *A store owner models the number of customers arriving at the store by Poisson distribution with unknown rate θ.*

Let X be the number of customers during one hour. If the data $X_1 = 3, X_2 = 4, X_3 = 2, X_4 = 2, X_5 = 3, X_6 = 5, X_7 = 6, X_8 = 1$ is observed, what is the MLE of θ?

Example 8. *Sampling from a Bernoulli distribution...*

Example 9. *Sampling from a normal distribution with a unknown mean...*