# HOMEWORK 0

STA 624.01, Applied Stochastic Processes
Spring Semester, 2015

**Due:** Tuesday, January 20th, 2015

**1** For every two events $A$ and $B$,
(a) prove the probability that exactly one of the two events will occur is

$$Pr(A) + Pr(B) - 2Pr(AB),$$

(b) show that

$$Pr(A) = Pr(AB) + Pr(AB^c).$$

**2** Suppose $X$ is a r.v. with $\mathbb{E}(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$. Show that

$$\mathbb{E}[X(X-1)] = \mu(\mu - 1) + \sigma^2.$$

**3** Suppose random variables $X$ and $Y$ are independent with finite mean such that

$$\mathbb{E}(X) = \mathbb{E}(Y).$$

Show that

$$\mathbb{E}[(X-Y)^2] = Var(X) + Var(Y).$$

**4** Suppose that the random variables $X_1, X_2, \cdots X_n$ are independent identically distributed from a uniform distribution on the interval $[0, 1]$. Let $Y_1 = \min\{X_1, X_2, \cdots X_n\}$ and $Y_2 = \max\{X_1, X_2, \cdots X_n\}$. Find $\mathbb{E}(Y_1)$ and $\mathbb{E}(Y_2)$. Show your work.

**5** Suppose that the random variables $X_1, X_2, \cdots X_n$ are independent identically distributed from a continuous distribution on the real line for which the p.d.f. is $f$. Find the expectation of the number of observations in the sample that fall within a specific interval $a \leq x \leq b$.

**Programming problem** Random walk on the finite set $\{-50, \ldots, 0, \ldots, 50\}$. You start from 0. With probability 1/2 to go $\pm 1$ each. Keep continuing this process until the number of moves becomes 1000. In addition please plot the values you take. Please write this code in `R`.