HOMEWORK 1

STA 624.01, Applied Stochastic Processes
Spring Semester, 2015

Due: Tuesday, January 27th, 2015

Readings: Chapter 1.1 to 1.4 of text.

Note: the computer problems require simulation and the use of a computer. You are allowed (encouraged, even) to use a computer in solving the other problems as well.

When giving numerical answers, please give results to four siginificant figures unless they are integer answers. So $1/2 = .5000$ for example. Also box your numerical answers.

Regular Problems

1 Do exercise 1.1 in the book.

2 Do exercise 1.2 in the book.

3 Do exercise 1.3 in the book.

4 Do exercise 1.4 in the book.

5 Do exercise 1.5 in the book.

Computer Problems

For this problem, please print out all code used and all results.

This Markov chain is called simple random walk with reflecting boundaries. The state space is $\{1, 2, \ldots, n\}$. It is defined as follows:

$$\begin{aligned}
P(X_{t+1} = i+1 | X_t = i) &= p, \ \forall i \in \{2, \ldots, n-1\} \\
P(X_{t+1} = i-1 | X_t = i) &= 1-p, \ \forall i \in \{2, \ldots, n-1\} \\
P(X_{t+1} = n-1 | X_t = n) &= 1-p \\
P(X_{t+1} = n | X_t = n) &= p \\
P(X_{t+1} = 1 | X_t = 1) &= 1-p \\
P(X_{t+1} = 2 | X_t = 1) &= p.
\end{aligned}$$

a) Write code for simulating this Markov chain.

b) Find the limiting distribution when $n = 10$ and $n = 15$ for $p = 0.2, 0.5, 0.6$ by simulating the chain multiple times starting from $X_0 = 1$.

c) For $n = 5$ with $p = 0.2, 0.5, 0.6$, estimate the expected number of steps needed to return to i starting at i for all $i \in \{1, 2, 3, 4, 5\}$.