HOMEWORK 3

STA 624.01, Applied Stochastic Processes
Spring Semester, 2015

Due: Thurs, February 26th, 2015

Readings: Chapter 2 of text,

Note: the computer problems require simulation and the use of a computer. You are allowed (encouraged, even) to use a computer in solving the other problems as well.

When giving numerical answers, please give results to four siginificant figures unless they are integer answers. So $1/2 = .5000$ for example. Also box your numerical answers.

Regular Problems

1 Gambler's Ruin, part I Suppose that a gambler plays a fair game so that at each play of the game she loses a \$1 with probability 1/2, and gains a \$1 with probability 1/2. The gambler starts with \$32 and stops when she reaches \$100 or is out of money.

a) What is the probability that the gambler ends up with \$100?

b) What is the expected number of plays before the gambler has either \$100 or \$0?

c) Now suppose the gambler starts with \$73. What is the probability that the gambler ends up with \$100?

2 Gambler's Ruin, part II Now suppose that the gambler is betting on red in American roulette. The chance of winning a \$1 is now 18/38 and losing a \$1 is 20/38.

a) What is the probability that the gambler ends up with \$100?

b) What is the expected number of plays before the gambler has either \$100 or \$0?

c) Now suppose the gambler starts with \$73. What is the probability that the gambler ends up with \$100?

3 Suppose that two unbiased coins are tossed repeatedly and after each toss the accumulated number of heads and tails that have appeared on each coin is recorded. Let X_n be the difference in the accumulated number of heads on coin A and coin B after the nth toss, i.e., $X_n =$ (Total number of heads on coin A) $-$ (Total number of heads on coin B). Thus, the state space $S = \{0, \pm 1, \pm 2, \cdots\}$. Show that the zero state, where the total number of heads equal on each coin, is null recurrent.

4 (Lawler 2.3) Consider the Markov chain with state space $\Omega = \{0, 1, 2, \ldots\}$ and transition probabilities

$$p(x, x+1) = 2/3;\ p(x, 0) = 1/3.$$

Show that the chain is positive recurrent and give the limiting probability π.

5 (Lawler 2.4) Consider the Markov chain with state space $\Omega = \{0, 1, 2, \ldots\}$ and transition probabilities

$$p(x, x+2) = p,\ p(x, x-1) = 1-p,\ x > 0.$$

$$p(0, 2) = p,\ p(0, 0) = 1-p.$$

For which values of p is this a transient chain?

Computer Problems

Simple Random Walk in $\mathbf{Z}^d$ Consider simple random walk on $\mathbf{Z}^1$, $\mathbf{Z}^2$, and $\mathbf{Z}^3$. (i.e., the transition probability $P(X_{n+1} = i+1 | X_n = i) = P(X_{n+1} = i-1 | X_n = i) = 1/2$ for $\mathbf{Z}^1$, and $P(X_{n+1} = i + e_j | X_n = i) = P(X_{n+1} = i - e_j | X_n = i) = 1/4$ for $\mathbf{Z}^2$, where e_j is the unit vector, and $P(X_{n+1} = i + e_j | X_n = i) = P(X_{n+1} = i - e_j | X_n = i) = 1/6$, where e_j is the unit vector, for $\mathbf{Z}^3$). For each of these, start at the origin and do the following. Estimate the expected distance aways from the origin after t steps, for t running from 1 to 100. Just use Euclidean distance, so in $\mathbf{Z}^3$, the distance of point (x, y, z) from the origin is $\sqrt{x^2 + y^2 + z^2}$. Conjecture a formula for this expected distance for $d = \{1, 2, 3\}$.