HOMEWORK 8

STA 624.01, Applied Stochastic Processes
Spring Semester, 2015

Due: Thursday, April 16th, 2015

Readings: Chapter 3 of text

Regular Problems

1 Birth and Death processes: Find an application of Birth and Death processes in your interested field. Summarize how it is applied. What are birth rates and death rates? What is the stationary distribution, if it is known? Why do they use Birth and Death processes?

2 Consider Yule process with the rate λ (i.e., it is a BD process with birth rate $x\lambda$ and death rate $\mu = 0$ for any state $x \in S$). Let $Y_x = \inf\{t : X_t = x\}$ for $x \geq 2$. Compute $E(Y_x)$.

3 For a Markov chain, the global balance equations $\pi_j = \sum_{i \in S} \pi_i P(i,j)$ for all j characterise the stationary distribution(s). The equations

$$\pi_i P(i,j) = \pi_j P(j,i) \text{ for all } i \text{ and } j$$

are called the local balance equations.

(a) Prove that if the local balance equations are satisfied, so are the global ones. Hence, if π is a distribution that satisfies the local balance equations, it is a stationary distribution. Hint: Note that, for all j, $\sum_{i \in S} P(j,i) = 1$.

(b) Find out the right way of formulating local balance equations in continuous time, that is, for Markov processes. Show that, again, local balance implies global balance.

Our conclusion is that local balance equations may be used to find stationary solutions–and in fact they typically yield simpler equations than what global balance does–but it is a method that does not always work.

4 Let X_t is distributed Yule process with the birth raye λ. Compute $\mathbf{E}(X_t)$ and $\mathbf{V}(X_t)$.

Computer Problem: M/M/1 queue This is a queue with one line. A customer arrives at the Poisson process with the rate $\lambda > 0$ and the first customer in the line is served at the rate $\mu > 0$. Customers are served with the exponential distribution with the rate $\mu > 0$.

In this computational problem, I want you to write a M/M/1 queue system and estimate the expected number of customers in the line at $t = 480$ minutes for

(a) $\mu = 1$ and $\lambda = 1$,
(b) $\mu = 1.5$ and $\lambda = 1$,
(c) $\mu = 1$ and $\lambda = 1.5$.