Name_____

STA 624 Midterm 1

Applied Stochastic Processes

March 10th, 2015

There are five questions on this test. DO use calculators if you need them. "And then a miracle occurs" is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 75 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: Modelling with Markov chains and processes

The self-fertilisingMarkov chain. Some plants such as rice and wheat have both male and female organs. These plants reproduce through self-fertilisation. Consider a particular genetic locus at which either of the two alleles may be A or a. The possible genotypes are thus AA, Aa and aa. Upon reproduction, each of the two new alleles is chosen at random from the two present ones. Let X_n be the genotype at generation n (X_n is thus AA, Aa or aa), motivate that $\{X_n\}$ is a Markov chain and write down its transition probability matrix and a transition graph.

Question 2: Computation

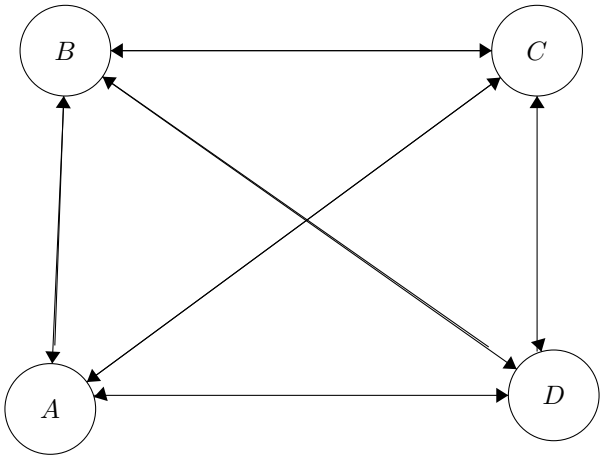
The concentration of a certain chemical compound outside a process industry is measured once per day and is then labelled as either normal (N) or high (H). The sequence of labels can be modelled by a discrete Markov chain. The probability that a normal value is followed by a high one is 0.1, and the probability that a high value is followed by a normal one is 0.95.

(a) Compute the probability that a high concentration is measured any given day.

(b) Compute the expected number of days during a period of 30 days, at which a high value is measured, but the following day shows a normal one.

Question 3: Absorption

A particle is placed at corner A in the picture below and then moves between the corners by taking steps in discrete time. In each step the particle chooses, with equal probabilities, one of the corners where it currently not located. All steps are made independently of each other. Compute the average number of steps the particle needs to take in order to visit all corners.



Questions 4: Examples (Note: you do not have to prove that your example meets the required criteria, you just have to present it.)

(a) Give an example of a 4 state Markov chain with period 3.

(b) Give an example of a null recurrent Markov chain.

(c) Give an example of a discrete time Markov chain whose stationary distribution is not unique.

(d) Give an example of a discrete time Markov chain which satisfies the GBE.

Question 5: Proof

Let X_n be a DTMC on a countable state S space and let $N(x) = \sum_{m=1}^{\infty} I_{\{X_m = x\}}$. Show $y \in S$ is recurrent if and only if $E[N(y)|X_0 = y] = \infty$.