

# HOMework 1

STA 624.01, Applied Stochastic Processes  
Spring Semester, 2016

**Due:** Thursday, Feb 4th, 2016

**Readings:** Chapter 1.1 to 1.4 of text.

Note: the computer problems require simulation and the use of a computer. You are allowed (encouraged, even) to use a computer in solving the other problems as well.

When giving numerical answers, please give results to four significant figures unless they are integer answers. So  $1/2 = .5000$  for example. Also box your numerical answers.

## Regular Problems

- 1 Do exercise 1.1 in the book.
- 2 Do exercise 1.2 in the book.
- 3 Do exercise 1.3 in the book.
- 4 Do exercise 1.4 in the book.
- 5 Do exercise 1.5 in the book.

### Computer Problems

For this problem, please print out all code used and all results.

This Markov chain is called simple random walk with reflecting boundaries. The state space is  $\{1, 2, \dots, n\}$ . It is defined as follows:

$$\begin{aligned}P(X_{t+1} = i + 1 | X_t = i) &= p, \forall i \in \{2, \dots, n - 1\} \\P(X_{t+1} = i - 1 | X_t = i) &= 1 - p, \forall i \in \{2, \dots, n - 1\} \\P(X_{t+1} = n - 1 | X_t = n) &= 1 - p \\P(X_{t+1} = n | X_t = n) &= p \\P(X_{t+1} = 1 | X_t = 1) &= 1 - p \\P(X_{t+1} = 2 | X_t = 1) &= p.\end{aligned}$$

- a) Write code for simulating this Markov chain.
- b) Find the limiting distribution when  $n = 10$  and  $n = 15$  for  $p = 0.2, 0.5, 0.6$  by simulating the chain multiple times starting from  $X_0 = 1$ .
- c) For  $n = 5$  with  $p = 0.2, 0.5, 0.6$ , estimate the expected number of steps needed to return to  $i$  starting at  $i$  for all  $i \in \{1, 2, 3, 4, 5\}$ .