Name____

STA 624 Midterm 1 Practice exam

Applied Stochastic Processes

March 8th, 2016

There are five questions on this test. DO use calculators if you need them. "And then a miracle occurs" is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 75 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: True or False

Mark whether each of the following states is true (T) or false (F). State a reason for each question.

(a) Any discrete time Markov chain with a finite state space is positive recurrent.

(b) A probability to return a null recurrent state from itself is less than 1.

(c) The expected time of visits at a null recurrent state is infinitely often.

(d) The expected time between visits at a null recurrent state is finite.

(e) A stationary distribution is a limiting distribution.

Question 2: Computation

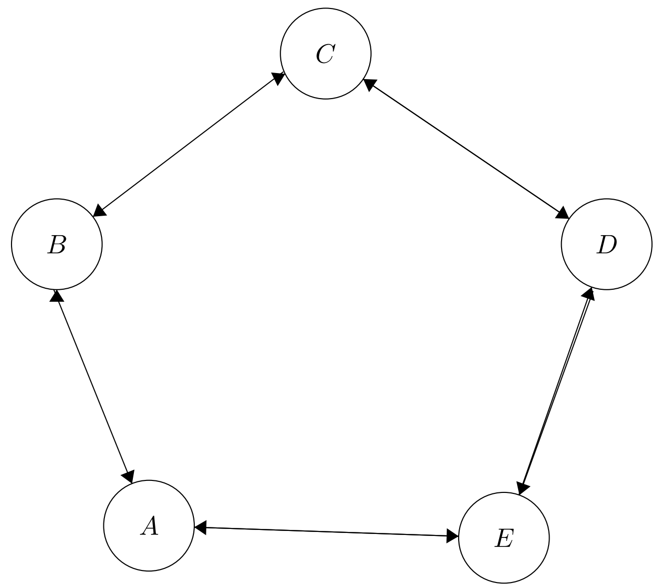
Cars, lorries and buses drive on a motorway, in one of the directions. It has been observed that after one out of ten cars follows a lorry, and after one out of ten cars follows a bus. After two out of three lorries follows another lorry, and after one out of three lorries follows a bus. After eight out of ten buses follows a car, and after one out of ten buses follows another bus.

(a) Suppose that the stream of cars, buses and lorries forms a Markov chain. Draw the transition graph and write down the transition probability matrix for this chain. Under stationarity, find out the fraction vehicles that are lorries.

(b) In practice, lorries often drive together in groups. Suppose instead that lorries always drive in groups of three, and that no cars or buses can sneak in between lorries. After such a group there is always a bus. Keeping the Markov assumption except for the groups of lorries, draw the transition graph and write down the transition probability matrix for this new chain. Under stationarity, find out the fraction vehicles that are lorries.

Question 3: Absorption

Five nodes are connected in a ring in the picture below. Two particles are placed at two adjacent nodes, and then move according to two independent random walks on this graph; at each step each particle moves either clockwise or counter-clockwise with equal probabilities, and independently of the other particle. Compute the average time it takes before the particles meet, i.e. are at the same node after a completed step.



Questions 4: Examples (Note: you do not have to prove that your example meets the required criteria, you just have to present it.)

(a) Give an example of a 5 state Markov chain with period 3.

(b) Suppose we have a branching process that a parent has no offspring with probability 1/4 and 2 offspring with probability 3/4. What is the extinction probability?

(c) Give an example of a 6 state Markov chain with four communication classes.

(d) Give an example of a Branching process with the extinct probability equal to 1.

Question 5: Proof

Let X_n be an irreducible aperiodic DTMC on a countable state space Ω and let $T_x = \min\{n \geq 1 | X_n = x\}$. Show $\mathbb{E}(T_x | X_0 = x) = 1/\pi(x)$ where π is the limiting distribution.