Name____

Practice STA 624 Exam 2

Applied Stochastic Processes

Spring, 2016

There are five questions on this test. DO use calculators if you need them. “And then a miracle occurs” is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 75 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: True or False

Mark whether each of the following states is true (T) or false (F). State a reason for each question.

(a) Suppose X_t is a continuous time Markov chain and Y_n is its underlying discrete time Markov chain. Then $P(Y_{n+1} = x | Y_n = x) = 0$ for any $x \in S$.

(b) In a birth and death process, $P(X_t = x + 1 | X_0 = x)$ for any $x \geq 0$ is distributed with Poisson distributions with rates λ_x and $P(X_t = x - 1 | X_0 = x)$ $x \geq 1$ is distributed with Poisson distributions with rates μ_x.

(c) Let $\{X_t, t \geq 0\}$ be Poisson processes with parameter λ. Define $Z_t = X_{k^2 \cdot t}$ where $k > 0$. Z_t is a Poisson process. If it is true then just state the rate.

(d) Let $\{X_t, t \geq 0\}$ and $\{Y_t, t \geq 0\}$ be independent Poisson processes with parameter λ and β. Define $\{Z_t, t \geq 0\}$ such that $Z_t = X_t - Y_t$. $\{Z_t, t \geq 0\}$ is a Poisson process. If it is true then just state the rate.

(e) For $M/M/\infty$ queue, there always exists a unique stationary distribution.

Questions 2: Examples (Note: you do not have to prove that your example meets the required criteria, you just have to present it.)

(a) Give an example of a continuous time Markov chain satisfying the detailed balance equations.

(b) Give an example of a positive recurrent birth-death chain

(c) Give an example of a transient M/M/7 queue.

(d) Give an example of a M/M/1 queue such that the expected value of the length of queue is 1/5.

Question 3: Modelling

Consider the following queueing system. Customers arrive to the system at constant intensity $0.5s^{-1}$ (that is, as a Poisson process). Every customer needs service from a server. Service times are random, but with a common distribution which is exponential with mean $3s$. If the server is busy (with another customer) at an arrival, the newly arrived customer must wait in a queue. Customers are served in the order they arrive, so each arriving customer must line up at the end of the queue.

(a) The service intensity is constant. Why? What is its value?

(b) Let X_t be the number of customers in the system at time t, including the one currently being served. Then $\{X_t\}_{t\geq 0}$ is a Markov process on the state space $\{0, 1, 2, \ldots\}$. Draw the transition graph for the embedded DTMC for this Markov process and write down the transition rates.

(c) Now assuming that the space for waiting customers is limited, so that only 4 customers are allowed to wait (thus not counting the one being served), and assuming that customers arriving to a full system are blocked (they cannot enter), how would you modify the transition graph for the embedded DTMC for this Markov process?

(d) Go back to (b) but assume that there are two identical servers, so that up to two customers may be served at the same time. How would you modify the model graph?

Question 4: Stationary distribution

A small elevator can carry at most two persons. The elevator goes from the ground floor to the first floor, and back again. The duration of such a round trip has an exponential distribution with mean $1/\mu$. People arrive to the elevator at the ground floor according to a Poisson process of rate λ where $\mu > \lambda$. Compute the stationary probability of n persons waiting at the ground floor.

Question 5: Theorem

Consider the continuous time Markov chain $\{X_t\}$ with a finite state space S. Let Q be the rate matrix for $\{X_t\}$ and $P(t)$ be the transition probability matrix for $\{X_t\}$. Prove Kolmogorov forward equation.