HOMEWORK 1

STA701.01, Statistical Inference
Fall Semester, 2013

Due: Thurs Sept 12th, 2013

1 The distribution of the number of failures y till the first success in independent Bernoulli trials, with probability of success π at each trial, is the geometric:

$$f(y) = (1-\pi)^y \pi, \; y = 0, 1, \cdots$$

Show that the geometric distribution is in the exponential family.

2 Consider the Gamma distribution with the pdf

$$f(y) = \frac{y^{-1}}{\Gamma(\nu)} \left(\frac{y\nu}{\mu} \right)^{\nu} \exp\left(-\frac{y\nu}{\mu} \right).$$

(a) Write down the likelihood equations for the estimation of ν and μ of the Gamma distribution.
(b) Show that the MLE of μ is $\hat{\mu} = \bar{y}$.

3 Consider the Inverse Gaussian distribution with the pdf

$$f(y) = \frac{1}{\sqrt{2\pi y^3}\sigma} \exp\left\{ -\frac{1}{2y} \left(\frac{y-\mu}{\mu\sigma} \right)^2 \right\}.$$

(a) Write down the likelihood equations for the estimation of μ and σ of the Inverse Gaussian distribution.
(b) Show that the MLE of μ is $\hat{\mu} = \bar{y}$.
(c) Show that the MLE of σ^2 is

$$\hat{\sigma}^2 = n^{-1} \sum_i \left(\frac{1}{y_i} - \frac{1}{\bar{y}} \right).$$