HOMEWORK 5

STA701.01, Statistical Inference
Fall Semester, 2013

Due: Thurs Oct 24th, 2013

1 (a) Suppose that we want to develop an informative prior distribution for the probability of observing heads when we flip a coin. Suppose that we think that the most likely probability of heads is 0.5 and that 0.75 would be "extreme." Find the parameters of a beta density so that the median is approximately 0.5 and the 0.9 quantile is 0.75.

(b) Suppose that we are going to flip a coin 20 times.

(i) Using a beta distribution, write down a prior density that describes your uncertainty about the probability of "heads."

(ii) Flip a coin 20 times and record the outcomes. Write down the likelihood function for the observed data.

(iii) Calculate the maximum likelihood estimate for the probability of "heads" and a 95% confidence interval.

(iv) Calculate the posterior distribution for the probability of "heads" and a 95% credible interval.

(v) Plot the log-likelihood function.

(vi) Plot the prior density.

(vii) Plot the posterior density.

(viii) Calculate the Bayes' factor comparing a uniform prior density to your informative prior density.

2 Show that the gamma distribution is the conjugate prior distribution for the mean of a Poisson likelihood.

3 Show that the gamma distribution is the conjugate prior distribution for the exponential likelihood.

4 Suppose we are using an Exponential(λ) distribution to model the lifetimes t_i of n items.

(a) Find the maximum likelihood estimator of λ.

(b) Assume n is large and find the standard error of the MLE of λ.

(c) Suppose that we observed $n = 50$ items and that

$$\sum_{i=1}^{50} t_i = 25.$$

Find a 90% confidence interval for λ.

(d) Suppose that $\lambda \sim \text{Gamma}(1, 2)$. Find the posterior distribution for λ.

(e) Suppose that we observed $n = 50$ items and that

$$\sum_{i=1}^{50} t_i = 25.$$

What is the posterior probability that λ falls in the 90% confidence interval found in (c)?