HOMEWORK 6

STA701.01, Statistical Inference
Fall Semester, 2013

Due: Thurs Oct 31st, 2013

1 Let $X_1, \ldots, X_n$ be a random sample from the Gaussian $N(\theta, \gamma^2)$, where γ^2 known, and let the prior distribution be the Gaussian $\theta \sim N(\phi, \tau^2)$. Suppose we have a quadratic loss function.

(a) Compute the Bayes estimator of θ and show that this estimator has constant risk as $\tau \to \infty$.

(b) Deduce the minimax estimator of θ and state its risk.

2 Events occur according to a Poisson process with intensity λ. Let n_t be the number of events observed at time t. The prior density of λ is

$$\pi(\lambda) = \frac{(\beta\lambda)^{\alpha-1} e^{-\beta\lambda} \beta}{\Gamma(\alpha)}$$

where $\alpha, \beta, \lambda > 0$. The loss function for estimating λ by d is $(d - \lambda)^2$.

(a) Derive the posterior distribution of λ given n_t.

(b) Find the Bayes estimator of λ. Suppose now that there is a cost ct associated with collecting observations for a period of time t. The combined cost of estimating λ by d is then $(d - \lambda)^2 + ct$.

(c) What is the prior expectation of this cost, anticipating the use of the Bayes estimator of λ?

(d) Show that, when $c > \alpha/\beta^2$, it is not worth collecting any observations.

(e) What is the optimal value of t when $c > \alpha/\beta^2$?