HOMEWORK 6

STA701.01, Statistical Inference
Fall Semester, 2013

Due: Thurs Nov 14th, 2013

1 Use the Neyman-Pearson lemma to find the form of the critical region for the best test of H_0 against H_1 when

(a) $X_1, X_2, \ldots, X_n$ are a random sample from a Poisson distribution with mean θ, and

$$H_0 : \theta = \theta_0, \text{ vs } H_1 : \theta = \theta_1,$$

where $\theta > 0$.

(b) $X_1, X_2, \ldots, X_n$ are a random sample from the exponential distribution with p.d.f.

$$f(x;\theta) = \begin{cases} \theta e^{\theta x}, & \text{if } x > 0, \\ 0 & \text{otherwise} \end{cases}$$

and

$$H_0 : \theta = \theta_0, \text{ vs } H_1 : \theta = \theta_1,$$

where $\theta_1 > \theta_0$.

(c) $X_{11}, X_{12}, \ldots, X_{1n_1} \sim N(\mu_1, \sigma_1^2)$ and $X_{21}, X_{22}, \ldots, X_{2n_2} \sim N(\mu_2, \sigma_2^2)$ all X_{ij} are indpendent of each other, σ_1^2 and σ_2^2 are known.

$$H_0 : \mu_2 = \mu_1, \text{ vs } H_1 : \mu_2 = \mu_1 + \delta,$$

where $\delta > 0$, μ_1 and δ are constants.

2 In Exercise 1(d), suppose that $\sigma_1^2 = \sigma_2^2 = \delta = 1$ and $n_1 = n_2 = n$. We wish to perform a best test with $\alpha = 0.01$. Find

(a) the power of the test when $n = 10$.

(b) the smallest value for n, for which we can achieve a power ≥ 0.95.

3 $X_1, X_2, \ldots, X_n$ are a random sample from a uniform distribution on $[0, \theta]$. Find the form of a best test of size α for $H0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, where $\theta_1 > \theta_0$. Suppose now that the alternative hypothesis is $H_1 : \theta > \theta_0$. Show that there exists a UMP test, and plot the power function of such a test.

4 A survey of the use of a particular product was conducted in four areas, with a random sample of 200 potential users interviewed in each area. The results were that in the four areas, respectively, x_1, x_2, x_3 and x_4 of the 200 interviewees said that they used the product. Construct an MLRT to test whether the proportion of the population using the product is the same in each area. Carry out the test, with $\alpha = 0.05$, when $x_1 = 76$, $x_2 = 53$, $x_3 = 59$, and $x_4 = 48$, using the large sample approximation for the distribution of the test statistic.