HOMEWORK 1

STA701.01, Statistical Inference
Fall Semester, 2014

Due: Thurs Sept 11th, 2014
Let $\mathcal{A}$ be the space of actions.

1 Read Section 1.4 on the book and do exercises 1 thru 5 on the section.

2 Two contestants simultaneously put up either one or two fingers. One of the players, call her player I, wins if the sum of the digits showing is odd, and the other player, player II, wins if the sum of the digits showing is even. The winner in all cases receives, from the loser, in dollars the sum of the digits showing.

1. Define $\Theta, \mathcal{A}$ and the loss function $l(\theta, a)$ for this game.
2. Define non-randomized actions.
3. Plot all randomized actions.

3 Let $\Theta = \mathcal{A} = \{1, 2\}$. Let

- Urn 1: 10 red balls, 20 blue balls, 70 green balls.
- Urn 2: 40 red balls, 40 blue balls, 20 green balls.

One ball is drawn from one of the two urns. Problem: decide which urn the ball came from if the loss function $L(\theta, a)$ are given by:

$\theta \backslash a$	1	2
1	0	10
2	6	0

Let $\delta = (\delta_R, \delta_B, \delta_G)$ with δ_X = probability of choosing urn 1 if color $X = x$ is observed. Calculate the risk function of such decision rules.

4 Suppose an unknown parameter θ is either 1/2 or 1/3. Our goal is to estimate θ with zero-one loss using the information from a single binary(θ) random variable X. Consider the following four non-randomized decision rules:

$$\begin{aligned}
\delta_1(X) &= 1/3 \\
\delta_2(X) &= 1/(3 - X) \\
\delta_3(X) &= 1/2 \\
\delta_4(X) &= 1/(2 + X).
\end{aligned}$$

1. Find the risk functions of each non-randomized decision rule (there are only two possible values of θ).
2. Let $a(X)$ be the probability to observe X. Compute randomized decision rules and calculate the risk function of such decision rules as a function of θ and $a(X)$.