HOMEWORK 2

STA701.01, Statistical Inference
Fall Semester, 2014

Due: Thurs Sept 18th, 2014
Let $\mathcal{A}$ be the space of actions.

1 Read Section 1.6 on the book and do exercise 3 on Section 1.5, exercises 1, 2, and 3 on Section 1.6, and exercises 1, 2, 3, 4 on Section 1.7.

2 Let $\Theta = \mathcal{A} = \{1, 2\}$. Let

- Urn 1: 10 red balls, 20 blue balls, 70 green balls.
- Urn 2: 40 red balls, 40 blue balls, 20 green balls.

One ball is drawn from one of the two urns. Problem: decide which urn the ball came from if the loss function $L(\theta, a)$ are given by:

$\theta \backslash a$	1	2
1	0	10
2	6	0

Let $\delta = (\delta_R, \delta_B, \delta_G)$ with δ_X = probability of choosing urn 1 if color $X = x$ is observed.

1. Calculate the risk function of such decision rules.
2. Plot the nonrandomized risk set S_0 and the risk set S.
3. Find the minimax.
4. Find the Bayes rule if we have the prior $\tau = (6/11, 1 - 6/11)$.

3 Suppose an unknown parameter θ is either $1/2$ or $1/3$. Our goal is to estimate θ with zero-one loss using the information from a single binary(θ) random variable X. Consider the following four non-randomized decision rules:

$$\begin{aligned} \delta_1(X) &= 1/3 \\ \delta_2(X) &= 1/(3 - X) \\ \delta_3(X) &= 1/2 \\ \delta_4(X) &= 1/(2 + X). \end{aligned}$$

1. Find the risk functions of each non-randomized decision rule (there are only two possible values of θ).
2. Plot the nonrandomized risk set S_0 and the risk set S.
3. Find the minimax.
4. Find the Bayes rule if we have the prior $\tau = (1/2, 1/2)$.

4 Suppose the parameter space Θ is finite. Show that the risk set S is a convex hull of all points in the nonrandomized risk set S_0.