HOMEWORK 3

STA701.01, Statistical Inference
Fall Semester, 2014

Due: Thurs Sept 25th, 2014 at 9:30am EST submitted via email.
Let $\mathcal{A}$ be the space of actions.

1 Read Section 1.8 and do exercises from 1 to 10 on Section 1.8.

2 Suppose we have a finite parameter space. Fix a prior distribution $\tau \in \Theta^*$ and if there exists a Bayes rule with respect to τ, show that there exists a nonrandomized Bayes rule with respect to τ.

3 Show that if the loss function is given as $L(\theta, a) = c(\theta - a)^2$ for $c > 0$, $\theta \in \Theta$, and $a \in (A)$, then the Bayes decision rule of θ is the mean of the posterior distribution of θ given the observation $X = x$.

4 Let $\Theta = \mathcal{A} = \mathbb{R}$. $L(\theta, a) = (\theta - a)^2$ and a prior distribution τ is a distribution for the normal with mean equal to zero and variance σ^2, $\sigma > 0$. Let the distribution of X given θ be a normal with mean θ and the variance equal one. Show that $d_\sigma(x) = \frac{x\sigma^2}{1+\sigma^2}$ is the extended Bayes rule.