

**HOMEWORK 4**  
STA701.01, Statistical Inference  
Fall Semester, 2014

**Due:** Thurs Oct 2nd, 2014 at 9:30am EST submitted via email.  
Let  $\mathcal{A}$  be the space of actions.

- 1 Do exercises from 1 to 6 on Section 2.1, exercises 1 to 3 on Section 2.2, exercises 1 to 3 on Section 2.3.
- 2 Find a counter example on Exercise 7 on Section 2.1 which is NOT on the solution set.
- 3 The risks for five decision rules  $d_1, \dots, d_5$  depend on the value of a positive-valued parameter  $\theta$ . The risks are given in the table below

	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$
$0 \leq \theta < 1$	10	10	7	6	8
$1 \leq \theta < 2$	8	11	8	5	10
$2 \leq \theta$	15	11	12	14	14

1. Which decision rules are at least as good as  $d_1$  for all  $\theta$ ?
  2. Which decision rules are admissible?
  3. Which is the minimax rule?
  4. Suppose  $\theta$  has a uniform distribution on  $[0, 5]$ . Which is the Bayes procedure and what is the Bayes risk for that rule?
- 4 Let  $X$  be uniformly distributed on  $[0, \theta]$  where  $\theta \in (0, \infty)$  is an unknown parameter. Let  $\mathcal{A} = [0, \infty)$  and the loss function  $L(\theta, a) = (\theta - a)^2$  where  $a \in \mathcal{A}$ . Consider the decision  $\delta_\mu(x) = \mu x$ , where  $\mu \geq 0$ . For what value of  $\mu$  is  $\delta_\mu$  unbiased? Show that  $\mu = \frac{3}{2}$  is a necessary condition for  $\delta_\mu$  to be admissible.