

HOMEWORK 5
STA703.01, Advanced Probability
Fall Semester, 2015

Due: Thurs October 8th, 2015

1 Let

$$f(x) = \frac{\sin x}{x}.$$

Show that the improper Riemann integral

$$\int_0^{\infty} \frac{\sin x}{x} dx$$

exists and it is $\frac{\pi}{2}$ but the Lebesgue integral of f on $[0, \infty)$ does not exist.

Note that in essence, improper Riemann integrals may exist because of fortunate cancellations, while the Lebesgue integral requires “absolute convergence.”

2 [Durrett, Exercises 1.6.1]

3 [Durrett, Exercises 1.6.2]

4 [Durrett, Exercises 1.7.1]

5 [Durrett, Exercises 1.7.3]