Name Solution

STA 320 Midterm 2

Probability

November 21st, 2008

There are five questions on this test. DO use calculators if you need them. "And then a miracle occurs" is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 50 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: Discrete probability distributions

If one-third of the persons donating blood at a clinic have O^+ blood, find the probability the following events:

(a) The first O^+ donor is the fourth donor of the day. Geom.

$$Y \sim Geom(\tfrac{1}{3})$$

$$P(Y=4) = \tfrac{1}{3}(1-\tfrac{1}{3})^{x-1} = 0.0988$$

(b) The second O^+ donor is the fourth donor of the day. Neg Binomial

$$Y \sim NegBin(2, \tfrac{1}{3})$$

$$P(Y=4) = \binom{4-1}{2-1}(\tfrac{1}{3})^2(1-\tfrac{1}{3})^{x-2}$$

$$= \binom{3}{1}(\tfrac{1}{3})^2(\tfrac{2}{3})^2 = 0.148$$

Question 2: Moment generating functions

Find the moment generating function for the Bernoulli random variable.

$$f(x) = \begin{cases} p & \text{if } x = 1 \\ 1-p & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$M(t) = \mathbb{E}(e^{tX}) = e^{t \cdot 0}(1-p) + e^{t1} p$$

$$= (1-p) + e^{t} \cdot p$$

Question 3: Continuous probability distributions

The number of defective circuit boards among those coming out of a solidering machine follows a Poisson distribution. For a particular 8-hour day, one defective bourd is found.

(a) Find the probability that it was produced during the first hour of operation for that day.

$$X \sim \text{unif}([0, 8])$$

$$P(X \leq 1) = \int_0^1 \frac{1}{8} dx = \boxed{\frac{1}{8}}$$

(b) Find the probability that it was produced during the last hour of operation for that day.

$$P(X \geq 7) = \int_7^8 \frac{1}{8} dx = \boxed{\frac{1}{8}}$$

Questions 4: Normal distribution

The weekly amount spent for maintenance and repair in a certain company has an approximately normal distribution with a mean of \$400 and a standard deviation of \$20. If \$450 is budget to cover repairs for next week, what is the probability that the actual costs will exceed the budget amount?

$X \sim N(400, 20^2)$

$Z = \frac{X-400}{20} \qquad Z \sim N(0,1)$

$P(X > 450) = P\left(\frac{X-400}{20} > \frac{450-400}{20}\right)$

$= P(Z > \frac{50}{20}) = P(Z > 2.5) = 1 - P(Z \leq 2.5)$

$= 1 - 0.9938$

$P(Z \leq 2.5) = 0.5 + 0.4938$

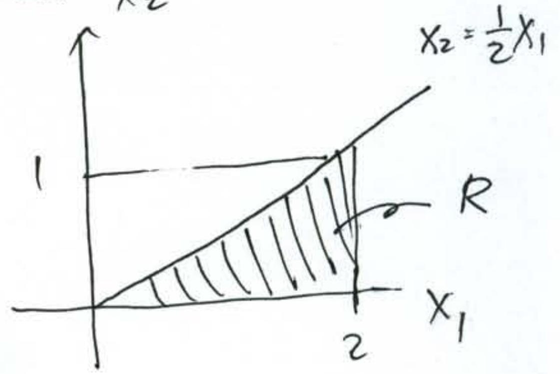
$\approx \boxed{0.0062}$

Questions 5: Bivariate probability distributions

An environmental engineer measures the amount (by weight) of particular pollution in air sample (of certain volume) collected over the smokestack of a coal-fueled power plant. Let X_1 denote the amount of pollutant per samplestack owhen a certain cleaning device on the stack is not operating and let X_2 denote the amount of pollutant per sample when the cleaning device is operating under similar environmental conditions. It is observed that X_1 is always greater than $2X_2$ and the relative frequency of (X_1, X_2) can be modeled by

$$f(x_1, x_2) = \begin{cases} k & \text{for } 0 \le x_1 \le 2,\ 0 \le x_2 \le 1,\ 2x_2 \le x_1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of k that makes a probability density function.



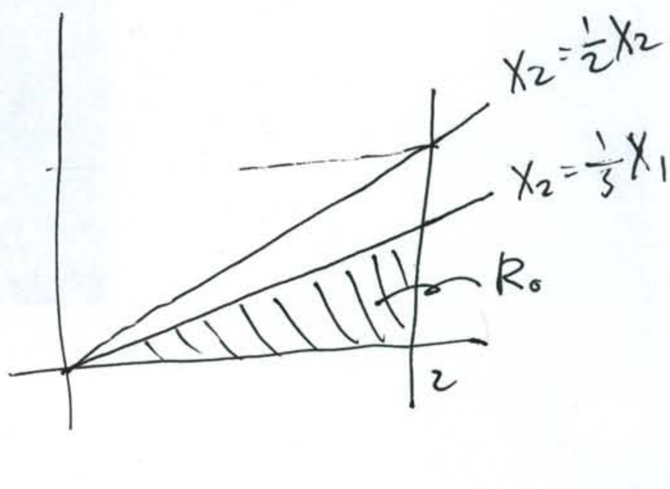
use

$$\iint_R dA = 1$$

$$\iint_R k\, dA = 1 \Rightarrow \boxed{k = 1}$$

(b) Find $P(X_1 \ge 3X_2)$.

Since This is unif,



$$P(X_1 \ge 3X_2) = \frac{\text{Area of } R_0}{\text{Area of } R} = \boxed{\frac{2}{3}}$$