Homework 5 Solutions

3.9 (a)

$$p(x) = \frac{\begin{pmatrix}\text{number of ways of}\\ \text{choosing } x \text{ from } 2\end{pmatrix}\begin{pmatrix}\text{number of ways of}\\ \text{choosing } 2-x \text{ from } 2\end{pmatrix}}{\begin{pmatrix}\text{total number of ways of}\\ \text{choosing a sample of } 2 \text{ from } 4\end{pmatrix}}$$

$$= \frac{\binom{2}{x}\binom{2}{2-x}}{\binom{4}{2}} \quad \text{for } x = 0,1,2$$

x	p(x)
0	1/6
1	2/3
2	1/6

(b)

$$p(x) = \frac{\binom{1}{x}\binom{3}{2-x}}{\binom{4}{2}} \quad \text{for } x = 0,1$$

x	p(x)
0	1/2
1	1/2

(c)

$$p(x) = \frac{\binom{0}{x}\binom{4}{0-x}}{\binom{4}{0}} \quad \text{for } x=0$$

$$P(x=0) = 1$$

(3.14) Let x be the mean age per group,

and use the percentage of drivers in each group.

x = mean age group	number of drivers	percent	x * percent	x^2	x^2 * percent
17	9.7	5.86	99.58	289	1692.81
22	16.9	10.21	224.52	484	4939.37
27	20.6	12.44	335.87	729	9068.48
32	20.5	12.38	396.14	1024	12676.33
37	18.6	11.23	415.58	1369	15376.45
42	16.1	9.72	408.33	1764	17150.00
47	12.6	7.61	357.61	2209	16807.61
52	10.2	6.16	320.29	2704	16655.07
57	9.5	5.74	326.99	3249	18638.59
62	9.3	5.62	348.19	3844	21587.68
67	8.3	5.01	335.81	4489	22499.21
72	13.3	8.03	578.26	5184	41634.78
	165.6	100.00	4147.16		198726.39

mean $\bar{x} = \dfrac{4147.16}{100} = 41.47$

Std dev $s = \sqrt{\left(\dfrac{198726.39}{100}\right) - (41.47)^2} = \sqrt{267.37} = 16.35$

median $\tilde{x} = 37$

(3.19) Let X = weekly number of breakdowns

From Tchebysheff, we have

$$P(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2} = .9$$

So $k = \sqrt{10}$, and the interval is

$$(\mu - k\sigma, \mu + k\sigma) = [4 - \sqrt{10}(.8), 4 + \sqrt{10}(.8)] = (1.47, 6.53)$$

(b) 8 breakdowns is $\frac{8-\mu}{\sigma} = \frac{8-4}{.8} = 5$ std. dev.'s from mean

so the interval $(\mu - 5\sigma, \mu + 5\sigma) = (4 - 5(.8), 4 + 5(.8))$

$= (0, 8)$

must contain $1 - \frac{1}{5^2} = .96$ of the probability.

So at most .04 of the probability mass can exceed 8 breakdowns, which is small, and so the director is safe in his claim.

3.24 (a) $P(X \le 6) = .250$

(b) $P(X \ge 12) = 1 - P(X \le 11) = 1 - .943 = .057$

(c) $P(X = 8) = P(X \le 8) - P(X \le 7) = .596 - .416 = .180$

3.26 Let X = number of surviving fish

X has binomial distribution with $n = 20$, $p = .8$

(a) $P(X = 14) = P(X \le 14) - P(X \le 13) = .196 - .087 = .109$

(b) $P(X \ge 10) = 1 - P(X \le 9) = 1 - .001 = .999$

(c) $P(X \le 16) = .589$