Homework 6 Solutions

3.48 (a) Let X = number of donors for the day until first O+ donor

Then X has geometric distribution with parameter $p = \frac{1}{3}$.

So $P(X=4) = \frac{1}{3}\left(\frac{2}{3}\right)^3 = \frac{8}{81} = .0988$

(b) Let Y = number of donors for the day until second O+ donor

Then Y has negative binomial distribution with parameters

$p = \frac{1}{3}$, $r = 2$.

So $P(X=4) = \binom{4-1}{2-1}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = 3\left(\frac{1}{9}\right)\left(\frac{4}{9}\right) = \frac{4}{27} = .1482$

3.53 (a) Let Y = number of customers it takes to sell 3 white appliances

Then Y has negative binomial distribution with $p = \frac{1}{2}$, $r = 3$.

So $P(Y=5) = \binom{5-1}{3-1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3} = 6\left(\frac{1}{8}\right)\left(\frac{1}{4}\right) = \frac{3}{16}$

(b) Let X = number of customers it takes to sell brown appliances

Then X has negative binomial distribution with $p = \frac{1}{2}$, $r = 3$,

So $P(X=5) = \binom{5-1}{3-1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3} = \frac{3}{16}$

(c) $P(Y=3) = \binom{3-1}{3-1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{3-3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

(d) P(all whites ordered before all browns)

$= P(Y \le 5) = P(Y=3) + P(Y=4) + P(Y=5)$

$= \frac{1}{8} + \binom{4-1}{3-1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) + \frac{3}{16} = \frac{1}{8} + \frac{3}{16} + \frac{3}{16} = \frac{1}{2}$

(3.54) (a) $P(Y=4) = p(4) = \frac{\lambda^4}{4!}e^{-\lambda} = \frac{2^4}{4!}e^{-2} = .0902$

(b) $P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - F(3) = 1 - .857 = .143$

(c) $P(Y < 4) = P(Y \leq 3) = F(3) = .857$

(d) $P(Y \geq 4 \mid Y \geq 2) = \frac{P(Y \geq 4, Y \geq 2)}{P(Y \geq 2)} = \frac{P(Y \geq 4)}{P(Y \geq 2)} = \frac{1 - P(Y \leq 3)}{1 - P(Y \leq 1)} = \frac{1 - F(3)}{1 - F(1)}$

$= \frac{1 - .857}{1 - .406} = .2407$

(3.55) Let Y = number of calls in one-minute period
Then Y has Poisson distribution with $\lambda = 4$.

(a) $P(Y=0) = p(0) = \frac{4^0}{0!}e^{-4} = e^{-4} = .0183$

(b) $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - F(1) = 1 - .092 = .908$

(c) Let X = number of calls in two-minute period
then X has Poisson distribution with $\lambda = 2(4) = 8$

So $P(X \geq 2) = 1 - F(1) = 1 - .003 = .997$

(3.61) Let Y = number of customer arrivals in a given hour
then Y has Poisson distribution with $\lambda = 8$

(a) $P(Y=8) = P(Y \leq 8) - P(Y \leq 7) = F(8) - F(7) = .593 - .453 = .140$

(b) $P(Y \leq 3) = F(3) = .042$

(c) $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - F(1) = 1 - .003 = .997$

3.67 $\mathbb{E}[Y(Y-1)] = \sum_{y=0}^{\infty} y(y-1) \cdot \frac{\lambda^y e^{-\lambda}}{y!}$

$$= \sum_{y=0}^{\infty} y(y-1) \cdot \frac{\lambda^2 \lambda^{y-2} e^{-\lambda}}{y(y-1)(y-2)!}$$

$$= \lambda^2 e^{-\lambda} \sum_{y=2}^{\infty} \frac{\lambda^{y-2}}{(y-2)!}$$

Let $x = y-2$

$$= \lambda^2 e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

$$= \lambda^2 e^{-\lambda} e^{\lambda}$$

$$= \lambda^2$$

$\mathrm{Var}[Y] = \mathbb{E}[Y^2] - [\mathbb{E}[Y]]^2$

$$= \mathbb{E}[Y^2] - \mathbb{E}[Y] + \mathbb{E}[Y] - [\mathbb{E}[Y]]^2$$

$$= \mathbb{E}[Y^2 - Y] + \mathbb{E}[Y] - [\mathbb{E}[Y]]^2$$

$$= \mathbb{E}[Y(Y-1)] + \mathbb{E}[Y] - [\mathbb{E}[Y]]^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$