Homework 9 Solutions

4.55 (a) $P(0 \le Z \le 1.2) = 0.3849$

(b) $P(-0.9 \le Z \le 0) = P(0 \le Z \le 0.9) = 0.3159$

(c) $P(0.3 \le Z \le 1.56) = P(0 \le Z \le 1.56) - P(0 \le Z \le 0.3)$
$= 0.4406 - 0.1179$
$= 0.3227$

(d) $P(-0.2 \le Z \le 0.2) = 2P(0 \le Z \le 0.2) = 2(0.0793) = 0.1586$

(e) $P(-2.00 \le Z \le 1.56) = P(0 \le Z \le 2.00) + P(0 \le Z \le 1.56)$
$= 0.4772 + 0.4406$
$= 0.9178$

4.56 (a) $z_0 = 0$

(b) $z_0 = 1.15$

(c) $z_0 = 1.19$

(d) $z_0 = -0.30$

(e) $z_0 = 1.645$

(f) $z_0 = 1.96$

4.61 Let X = resistance of wires produced by Company A. Then X has a normal distibution with parameters $\mu = 0.13$ and $\sigma = 0.005$.

(a) $P(0.12 \le X < 0.14) = P\left(\frac{0.12-0.13}{0.005} < \frac{X-0.13}{0.005} < \frac{0.14-0.13}{0.005}\right)$

$= P(-2 < Z < 2)$

$= 2P(0 < Z < 2)$

$= 2(0.4772) = 0.9544$

(b) Let Y = number of wires of a sample of four from Company A that meet specifications.

Then $Y \sim \text{Binomial}(n=4, p=0.9544)$.

$$P(Y=4) = \binom{4}{4}(0.9544)^4(1-0.9544)^0 = 0.8297$$

4.69 (a) Yes, it does appear that the total points can be modeled by a normal distibution.

(b) According to the empirical rule, 68% of the data should lie one standard deviation above and below the mean and 95% of the data should lie within two standard deviations above and below the mean. Hence, consider the interval $(\bar{x}-s, \bar{x}+s) = (143-26, 143+26) = (117, 169)$. Notice that more than 77% of the games had total scores within $(117, 169)$. Now consider the interval $(\bar{x}-2s, \bar{x}+2s) = (143-2(26), 143+2(26)) = (91, 195)$. Notice that less than 5% of the total scores fall outside of this region.

(c) No and no. A score of 200 is greater than two standard deviations away from the mean. Such a score should occur less than 2.5% of the time, according to the empirical rule. A score of 250 is greater than three standard deviations away from the mean, making it even less likely to occur.

(d) About 4 games.

4.73 (a) $1 = \int_0^1 kx^3(1-x)^2\,dx = k\int_0^1 x^{(4-1)}(1-x)^{(3-1)}\,dx$

$$= k\,\frac{\Gamma(4)\Gamma(3)}{\Gamma(7)} = k\,\frac{(6)(2)}{720} = k\left(\frac{1}{60}\right)$$

So $k = \dfrac{\Gamma(7)}{\Gamma(4)\Gamma(3)} = 60$, and $X \sim \text{Beta}(\alpha = 4, \beta = 3)$.

(b) $E(X) = \dfrac{\alpha}{\alpha+\beta} = \dfrac{4}{7}$

$$V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{4(3)}{(7)^2(8)} = \frac{3}{98}$$

4.74 $E(X^2) = \int_0^1 x^2\,\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,x^{\alpha-1}(1-x)^{\beta-1}\,dx$

$$= \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,x^{\alpha+2-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(\alpha+2+\beta)}\int_0^1 \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+2)\Gamma(\beta)}\,x^{\alpha+2-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(\alpha+2+\beta)}$$

$$= \frac{\Gamma(\alpha+\beta)(\alpha+1)\alpha\Gamma(\alpha)}{\Gamma(\alpha)(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)}$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$$

We know $E(X) = \dfrac{\alpha}{\alpha+\beta}$

Then $V(X) = E(X^2) - (E(X))^2 = \dfrac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \left(\dfrac{\alpha}{\alpha+\beta}\right)^2$

$$= \frac{\alpha(\alpha+1)(\alpha+\beta) - \alpha^2(\alpha+\beta+1)}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$