

HOMEWORK 10

STA5724.01, Probability

Fall Semester, 2007

Due: Friday, November 16th, 2007

1 Suppose $Var(X) < \infty$ and $Var(Y) < \infty$. Show that $Cov(X, Y) < \infty$.

2 Suppose that the distribution of X is symmetric with respect to the point $x = 0$. Also $0 < \mathbb{E}(X^4) < \infty$ and $Y = X^2$. Show that X and Y are uncorrelated.

3 Suppose $0 < \sigma_X^2 < \infty$ and $0 < \sigma_Y^2 < \infty$ and suppose $U = aX + b$ and $V = cY + d$, where $a, b, c, d \in \mathbb{R}$ are constant with $a \neq 0$ and $c \neq 0$. Show that $\rho(U, V) = \rho(X, Y)$ if $ac > 0$, and $\rho(U, V) = -\rho(X, Y)$ if $ac < 0$.

4 Suppose $Cov(X, Z)$ and $Cov(Y, Z)$ exist. Show that

$$Cov(aX + bY + c, Z) = aCov(X, Z) + bCov(Y, Z)$$

where $a, b, c \in \mathbb{R}$ are constant.

5 Suppose that $Var(X) = Var(Y)$ and $Var(X + Y) < \infty$ and $Var(X - Y) < \infty$. Show that $X + Y$ and $X - Y$ are uncorrelated.