HOMEWORK 11

STA5724.01, Probability
Fall Semester, 2007

Due: Friday, November 30th, 2007

1 For each nonnegative integer n, let X_n be a nonnegative r.v. with finite mean μ_n. Prove that if $\lim_{n\to\infty} \mu_n = 0$ then $\lim_{n\to\infty} Pr(|X_n - 0| < \epsilon) = 1$ for any $\epsilon > 0$.

2 Suppose that X is a r.v. with $\mathbb{E}(X) = \mu$ and $\mathbb{E}[(X-\mu)^4] = \beta$. Prove

$$Pr(|X - \mu| \geq t) \leq \beta/t^4, \ \forall t > 0.$$

3 Suppose X is a r.v. with $\mathbb{E}(X^k)$ exits for any positive integer k and $Pr(X \geq 0) = 1$. Prove that for $k > 0$ and $t > 0$

$$Pr(X \geq t) \leq \frac{\mathbb{E}(X^k)}{t^k}.$$

4 Suppose X_t and Y_t are two independent Poisson processes with rate parameters λ_1 and λ_2, respectively, measuring of calls arriving at two different phones. Let $Z_t = X_t + Y_t$. Show that Z_t is a poisson process. What is the rate parameter for Z?

5 Let X_t and Y_t be two independent Poisson processes with rate parameters λ_1 and λ_2, respectively, measuring the number of customers in store 1 and store 2, respectively. What is the probability that a customer arrives in store 1 before any customers arrive in store 2?

6 If the m.g.f. of a r.v. X is $\psi(t) = \exp(t^2)$ for $-\infty < t < \infty$, what is the distribution of X?