HOMEWORK 1

STA5724.01, Probability

Fall Semester, 2007

Due: Friday, August 31, 2007

1 For events A, B such that $A \subset B$, show that

$$B^c \subset A^c.$$

For every three events A, B, C, show that

$$A(B \cup C) = (AB) \cup (AC).$$

2 For every two events A and B, show that

$$(A \cup B)^c = A^c \cap B^c$$

and

$$(A \cap B)^c = A^c \cup B^c.$$

3 For every three events A, B, C, show that

$$A - (B \cup C) = (A - B) \cap (A - C)$$

and

$$A - (B \cap C) = (A - B) \cup (A - C)$$

4 For every two events A and B, show that AB and AB^c are disjoint and also prove that

$$A = (AB) \cup (AB^c).$$

In addition show that AB, AB^c, and A^cB are disjoint.

5 For every collection of events A_i for $i \in I$ (I is an index set), show that

$$(\cup_{i \in I} A_i)^c = \cap_{i \in I} A_i^c$$

and

$$(\cap_{i \in I} A_i)^c = \cup_{i \in I} A_i^c$$

6 Let A_i, for $i = 1, 2, \cdots$, be an infinite sequence of events. For $n = 1, 2, \cdots$, let $B_n = \cup_{i=n}^{\infty} A_i$ and let $C_n = \cap_{i=n}^{\infty} A_i$. Show that $B_1 \supset B_2 \supset \cdots$ and $C_1 \subset C_2 \subset \cdots$.