HOMEWORK 2

STA5724.01, Probability

Fall Semester, 2007

Due: Friday, September 7, 2007

1 Prove that for every two events A and B, the probability that exactly one of the two events will occur is

$$Pr(A) + Pr(B) - 2Pr(AB).$$

For every two events A and B, show that

$$Pr(A) = Pr(AB) + Pr(AB^c).$$

2 Let A_i, for $i = 1, 2, \cdots$, be an arbitrary infinite sequence of events and let B_j, for $j = 1, 2, \cdots$, be another arbitrary infinite sequence of events defined as follows: $B_1 = A_1$, $B_2 = A_1^c A_2$, $B_3 = A_1^c A_2^c A_3, \cdots$. Prove that

$$Pr\left(\cup_{i=1}^n A_i\right) = \sum_{i=1}^n Pr(B_i), \text{ for } n = 1, 2, \cdots$$

and

$$Pr\left(\cup_{i=1}^\infty A_i\right) = \sum_{i=1}^\infty Pr(B_i).$$

3 For every events A_i, for $i = 1, 2, \cdots, n$, prove that

$$Pr\left(\cup_{i=1}^n A_i\right) \leq \sum_{i=1}^n Pr(A_i).$$

4 Consider an experiment in which a fair coin is tossed once and a balanced die is rolled once.

a. Describe the sample space for this experiment.

b. What is a probability that a head will be obtained on the coin and an odd number will be obtained on the die?

c. If 12 balls are thrown at random into 20 boxes, what is the probability that no box will receive more than one ball?

5 Prove that for all positive integer n

$$\sum_{i=0}^n \binom{n}{i}^2 = \binom{2n}{n}.$$

6 **a.** Prove for all positive integer n

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n.$$

b. Prove for all positive integer n

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0.$$