

HOMEWORK 3  
STA5724.01, Probability  
Fall Semester, 2007

**Due:** Friday, September 14, 2007

**1** Prove that the Binomial theorem. Hint: see page 35 in the text book..

**2** A restaurant has  $n$  items on its menu. During a particular day,  $k$  customers will arrive and each one choose one item. The manager wants to count how many ways of customer's choices are possible without regarding the order in which the choices are made. (For example, if  $k = 3$  and  $a_1, a_2, \dots, a_n$  are the items in the menu, then  $a_1 a_3 a_2$  is not different from  $a_1 a_2 a_3$ .) Prove that the number of ways of customer's choices is  $\binom{n+k-1}{k}$ .

**3** Prove the multinomial theorem. Hint: see page 38 in the text book.

**4** Let  $A_1, A_2, \dots$  be an infinite sequence of events such that  $A_1 \subset A_2 \subset \dots$ . Prove that

$$Pr(\cup_{i=1}^{\infty} A_i) = \lim_{i \rightarrow \infty} Pr(A_i).$$

Hint is on page 44 in the book.

**5** Let  $A_1, A_2, \dots$  be an infinite sequence of events such that  $A_1 \supset A_2 \supset \dots$ . Prove that

$$Pr(\cap_{i=1}^{\infty} A_i) = \lim_{i \rightarrow \infty} Pr(A_i).$$

Hint is on page 44 in the book.

**6** Let  $A_i$ , for  $i = 1, 2, \dots, n$ , be an arbitrary finite sequence of events. Show that the probability that exactly one of these  $n$  events will occur is

$$\sum_{i=1}^n Pr(A_i) - 2 \sum_{i < j} Pr(A_i A_j) + 3 \sum_{i < j < k} Pr(A_i A_j A_k) - \dots + (-1)^{n+1} n Pr(A_1 A_2 \dots A_n).$$