HOMEWORK 4

STA5724.01, Probability
Fall Semester, 2007

Due: Friday, September 21, 2007

1 For any events A, B, and C, such that $Pr(C) > 0$, prove that

$$Pr(A \cup B|C) = Pr(A|C) + Pr(B|C) - Pr(AB|C).$$

2 Assuming that A and B are independent events, prove that A^c and B^c are also independent events.

3 Suppose A, B, and C are three independent events such that $Pr(A) = 1/4$, $Pr(B) = 1/3$, $Pr(C) = 1/2$.
(a) Determine the probability that none of the three events will occur.
(b) Determine the probability that exactly one of these three events wull occur.

4 Prove the following statement:
Let $A_1, \cdots, A_k$ be events such that $Pr(A_1 \cdots A_k) > 0$. Then, $A_1, \cdots, A_k$ are independent if and only if for every two disjoint subsets $\{i_1, \cdots, i_m\}$ and $\{j_1, \cdots, j_l\}$ of $\{1, \cdots, k\}$ we have

$$Pr(A_{i_1} \cdots A_{i_m} | A_{j_1} \cdots A_{j_l}) = Pr(A_{i_1} \cdots A_{i_m}).$$

5 Prove the following statement:
Suppose A_1, A_2, and B are events such that $Pr(A_1 B) > 0$. Then, A_1 and A_2 are conditionally independent given B if and only if $Pr(A_2|A_1 B) = Pr(A_2|B)$.